Electrical Technology
for
Telecommunications

BY

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PREFACE

THIS volume has been written particularly for the use of first- and second-year telecommunication students. In this new edition a number of examples have been added. Thanks are due to the City and Guilds of London Institute, Department of Technology, for permission to reproduce questions set in their examinations.

The majority of books on electrical technology treat at some length with the construction and characteristics of machines and with armature windings. This has been omitted and emphasis has been laid on the alternating current circuit, the transformer, instruments and measurements. The book covers the syllabus of the City and Guilds examination in Telecommunications (Principles) I and, in conjunction with the author's *Second Year Radio Technology*, the syllabuses for the examinations in Telecommunications (Principles) II and Radio I.

Thanks are due to Dr. A. Morley and Dr. E. Hughes for permission to make use of material from their books, "Elementary Engineering Science" and "Electrical Engineering Science."

W. H. D.

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CHAPTER I

THE ELECTRIC CURRENT

Effects of the Electric Current.

When a quantity of electricity is set in motion certain well defined effects are observable. The most important of these effects are:

(1) The heating effect.
(2) The chemical effect.
(3) The magnetic effect.

The Heating Effect.

A moving vehicle is brought to rest by means of a brake due to the friction of a band pressing against a drum. If the surface of the brake-drum be touched by the hand it will be found to be appreciably hot. The energy of the moving vehicle in being brought to rest has been converted into heat. Something analogous to friction takes place when there is a flow of electricity inside a wire. The wire gets hot and the energy expended in driving the electricity through the wire has been converted into heat. It is important to note that no energy has been lost or destroyed; the whole of the electrical energy has been transformed into energy in the form of heat.

The practical applications of the heating effect are numerous.

To mention a few examples, there are incandescent electric lamps, cookers, radiators, soldering irons. Special applications of the heating effect are the fuse and the hot-wire measuring instrument.

In the fuse, which is a fine bare wire of tin or copper, if the strength of the current by some means exceeds the safe value the heat produced is sufficient to melt the wire and the flow of electricity is stopped.

In the measuring instrument the current passes through a wire which gets warm and expands. The expansion is taken up and communicated to a pointer moving over a

scale. The deflection of the pointer is therefore a measure of the strength of the current.

The amount of heat produced by a given current is quite independent of the direction of flow. Electricity which is set in motion by a battery or from a direct current (d.c.) supply always flows in one direction. On the other hand, electricity set in motion by an alternating current (a.c.) supply reverses in direction periodically. In the case of a public supply (a.c.) the direction of flow is reversed every $\frac{1}{100}$th part of a second so that a complete reversal back to the original direction will take $\frac{2}{100}$ or $\frac{1}{50}$th part of a second. In the case of alternating currents, as used in radio transmitters, a complete reversal may take place in $\frac{1}{1,000,000}$th part of a second.

However quickly the reversals or alternations may be taking place the heating effect is the same, and in consequence the hot-wire instrument will be found in use for the measurement of currents in radio transmitting circuits.

The Chemical Effect.

If a current of electricity be made to pass through a solution of copper sulphate by means of two plates immersed in the liquid it is found that one plate increases in weight due to copper being deposited. If the direction of flow of electricity be reversed the other plate will receive a coating of copper. It has been agreed that the plate which receives the deposit is the one where the current is leaving the liquid. Similarly, a current passed through a solution of silver nitrate will deposit a coating of silver on the plate where the current is leaving. Practical applications of this effect are found in all kinds of electro plating, such as silver, gold, nickel, cadmium, etc.

It will be appreciated that an alternating current, which reverses its direction of flow periodically, cannot be employed to utilise the chemical effect since the two plates will alternately receive and lose the deposit of metal. Owing to the accuracy with which the weight of metal deposited can be determined, the chemical effect has been accepted internationally as the standard of measurement for quantity of electricity.

Unit quantity of electricity is that amount which will deposit 0·00033 gramme of copper or 0·001118 gramme of silver. This unit is called a *coulomb*.

An electric current is the rate of flow of electricity and may be compared with the flow of water in a pipe. Thus, if a quantity of water measured at 50 gallons passes a given point in the pipe in 10 seconds, the rate of flow is 5 gallons per second. Similarly, if a quantity of electricity measured at 5000 coulombs passes in 1000 seconds, the rate of flow is 5 coulombs per second. A quantity of electricity of 5000 coulombs would deposit 5000 × 0·00033 or 1·65 grammes of copper and 5000 × 0·001118 or 5·59 grammes of silver.

Rate of flow of electricity is of such importance that the coulomb per second has been given a name—the *ampere*. Thus, in the above example, electricity is said to flow at the rate of 5 amperes, or 5A, the letter "A" being the symbol for "ampere." Sometimes, as in radio and telephony circuits, currents are met with having values less than one ampere. It is then customary to speak of a current of say 0·01 ampere as 10 milliamperes, or 10 mA, the prefix "milli" (denoted by "m") meaning a thousandth part.

Since 1 ampere is 1 coulomb per second, it follows that unit quantity of electricity may also be called an *ampere-second*. For many purposes, such as for the capacity rating of an accumulator, the coulomb, or ampere-second, is inconveniently small and a larger unit called the ampere-hour (denoted by Ah) is used. One ampere-hour is equal to 3600 coulombs. An accumulator having a capacity of 60 Ah will deliver a current of 6 amperes for 10 hours or 3 amperes for 20 hours.

The Magnetic Effect.

As is well known a short pivoted magnet will set itself along a definite direction with respect to the surface of the earth. This is the compass needle. If now, a wire carrying a current be placed above the needle and in line with it, the needle will be deflected from its original direction. This shows that the current is producing a magnetic effect in the vicinity of the wire and when the

. . . . be reversed the deflected in the opposite direction. Thus, by thi the direction of flow of a direct current may be If the wire is carrying an alternating current the needle will not be deflected since it is experienc and opposite forces as the current passes first in then in the other direction.

The magnetic effect produced by a current increased by winding the wire in the form of a co times known as a *solenoid*. If the coil has bee on a bobbin with a hollow centre it will be foun rod of iron is attracted inside the bobbin with an ap force. This force acts on the iron independentl direction of flow and so may be used for the meas of both direct and alternating currents. Such an ins is known as a *moving iron ammeter*.

Other practical applications of the magneti among many, are the electric bell, the telephone the relay.

CHAPTER II

THE ELECTRIC CIRCUIT

Conductors and Insulators.

Some substances will allow of the passage of an electric current much more readily than others. In this connection, all the metals and metallic alloys offer but little opposition compared with most non-metallic substances. The former are referred to as *conductors* and the latter as *insulators.* It must be remembered, however, that these terms are only relative. There is no perfect conducting material and no perfect insulating material. Metals such as silver and copper are very good conductors, whereas materials like silk, rubber, glass, porcelain and the plastics, such as bakelite, are good insulators.

Water with metallic salts in solution is also a fair conductor, but the mineral and vegetable oils are good insulators. When it is desired to convey a quantity of electricity from one point to another with little loss of energy, copper is used as the conductor, and to minimize leakage to neighbouring conductors, such as the earth, the copper wire or cable is covered with an insulating material such as rubber or paper. Where the conductor is run overhead as in transmission lines and wireless aerials, the metal may be bare, since air is an insulator, and porcelain insulators employed at the supporting masts or towers.

Conditions for Current Flow.

In order that a flow of electricity may take place two conditions must be satisfied. These are:

(1) The circuit to convey the current must be conducting throughout.

(2) A source of electrical pressure must be applied to the circuit.

The first condition has already been discussed and the two well-known sources of electrical pressure are the *cell* and the electrical generator or *dynamo.*

The Chemical Cell.

Various substances when immersed in certain conducting liquids will produce an electrical pressure or, as it is more usually termed, *electromotive force* (abbreviated as e.m.f.).

The common Leclanché cell consists of plates or rods of zinc and carbon immersed in a solution of ammonium chloride, while the lead accumulator consists of plates of lead and lead peroxide immersed in dilute sulphuric acid. The conducting liquid (sometimes a paste is used, as in the so-called "dry" type of Leclanché cell) employed in chemical cells is called the *electrolyte*. The flow of electricity set in motion by the electromotive force of a cell is unidirectional, the plate at which the current leaves the cell being termed the *positive plate* and the plate at which the current enters the cell after traversing the external conducting circuit being termed the *negative plate*. A cell is

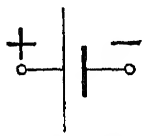

FIG. 1.—A cell.

represented diagrammatically as in Fig. 1, the longer thin line being the positive plate (marked +) and the shorter thick line the negative plate (marked —).

In the Leclanché cell the carbon rod is the positive and may be recognised as the terminal situated in the centre of the cell. In the accumulator the positive terminal is painted red or a + sign marked close by, the negative terminal being painted black or marked with a — sign. As the chemical energy, represented by the various ingredients in the cell, is used up the electromotive force of the cell slowly falls and finally the cell is of no further use. With the *primary cell*, such as the Leclanché, this means that fresh ingredients must be added or the cell discarded as is usually the case with the dry type but with the *secondary cell*, such as the lead accumulator, the ingredients may be restored by utilising the chemical effect produced by sending a current through the electrolyte in

the opposite direction to that which obtained when the cell was being used as a source of e.m.f. This is known as charging the accumulator and means that the positive terminal of the charging supply must be connected to the positive terminal of the accumulator.

The Standard Cell.

In order to compare the magnitudes of different e.m.f.'s it is necessary to have a standard of reference and it has been agreed internationally that a primary cell made up to a certain specification, and known as a Weston or cadmium cell, shall be that standard. Unfortunately, the magnitude of the e.m.f. of the standard cell is not exactly one unit but 1·0183 units. The name given to the unit of e.m.f. is the *volt* (denoted by the letter V).

It must be appreciated that the standard cell is not used as a source of electrical energy but solely as a standard of comparison with other e.m.f.'s. In practice, instruments known as *voltmeters* and identical in construction with ammeters are used for the measurement of electrical pressures. These instruments have scales marked in volts and have been calibrated against accurate voltmeters which, in turn, have been calibrated against the standard cell.

The e.m.f. of the Leclanché cell is nominally 1·5 volts, that of the lead accumulator 2 volts and public supply a.c. mains have been standardised at 240 volts, although variations in pressure between different areas do still occur.

Cells in Series.

For many purposes the e.m.f. of a single cell is too small and so a number of cells may be connected together to form a *battery*. This is shown in Fig. 2, where it will

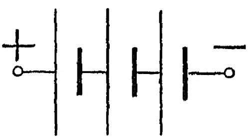

FIG. 2.—Cells in series.

be seen that the positive of one cell is connected to the negative of the next, and so on. This is known as a *series* connection and the total e.m.f. supplied by the battery is now equal to the sum of the individual e.m.f.'s of the various cells. Thus, fifty cells of the lead accumulator type connected in series would give a nominal e.m.f. of 100 volts and a " dry " battery of eighty cells would give an e.m.f. of 120 volts. Since the same value of current passes through each cell in turn the capacity of a battery in ampere-hours is no greater than that of a single cell, but the amount of power and energy available has been multiplied by the number of cells in the battery.

Cells in Parallel.

The magnitude of the current which a cell or generator may be allowed to pass through the electrolyte or windings is strictly limited by its physical dimensions. A current greater than the manufacturer's rated value will produce excessive heating, so that if the desired value of current is in excess of the rating for the individual cell or generator a number may be connected in *parallel* as shown in Fig. 3. Here, all the positive terminals are connected together forming a common terminal and similarly with the negative terminals. The e.m.f. of the battery, or combination of generators, is now the same as that of an individual cell or generator.

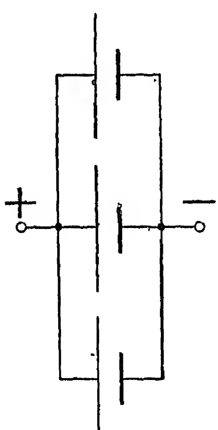

FIG. 3.—Cells in parallel.

The parallel arrangement is seldom met with in connection with cells but is common practice for generators in power stations where, as the demand for energy, and hence current, increases at certain periods of the day, one or more extra generators are connected in parallel with those already running. If the cells or generators are of the same size the current in the external circuit is shared equally. Thus in Fig. 3 if the

current being circulated in the external circuit is 15 amperes the current passing through the electrolyte of each cell will be 5 amperes.

The Electric Circuit.

Fig. 4 shows a simple circuit where the source of e.m.f. is a cell which is connected to a lamp L. A switch S controls the starting and stopping of the current, an ammeter A registers the value of the current and a voltmeter V the value of the pressure applied to the external circuit from the cell. The arrows indicate the conventional direction of flow of current when the switch contacts are closed. It will be noted that the ammeter is connected in series so that the current passing through the lamp filament also passes through the ammeter. The voltmeter, on the other hand, is connected in parallel so that the current passing through this instrument is quite independent of the current through the lamp and is solely determined by the electrical pressure of the cell. The voltmeter is so designed that the current required to deflect the needle is only a few milliamperes.

It must be appreciated that the cell or generator does not produce electricity but is necessary to keep it in motion. In the same way, a pump does not manufacture water but is used to provide a pressure which causes the water to flow through the pipe.

CHAPTER III

ELECTRICAL RESISTANCE

Pressure and Current.

Referring to the circuit of Fig. 4, suppose the reading of the voltmeter is 2 volts and that of the ammeter 1 ampere. Now add another cell in series so that the reading of the voltmeter is 4 volts. It will be found that the

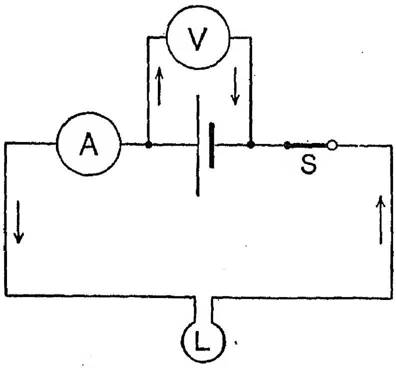

FIG. 4.—A simple circuit.

current has increased to 2 amperes and if the e.m.f. of the battery be still further increased to 6 volts the current will then be 3 amperes. In other words, the current passing in a given circuit is proportional to the e.m.f. applied to the circuit. Let E represent the e.m.f. and let I represent the current, then we may write in symbol form :

$$I \propto E.$$

It follows from this that for any given circuit $\frac{E}{I}$ is a constant, and for the circuit of Fig. 4 is 2.

Resistance.

The constant referred to above is termed the *resistance* of the circuit and is measured in units called *ohms* (denoted

by the symbol Ω, pronounced "omega"). Thus, if the e.m.f. applied to a circuit is 1 volt and the current is 1 ampere, that circuit is said to have a resistance of 1 ohm. In general, let R represent the resistance of the circuit then $R=\frac{E}{I}$.

It will be appreciated that different circuits will possess different resistances. Thus, an ordinary electric lamp of the "60-watt size" designed for use on a 230-volt supply will possess a resistance of the order of 900 ohms. The resistance of the heater of a valve as used in radio receivers and rated at 4V, 1A will be 4 ohms. A length of copper wire will possess some resistance although it may be only a small fraction of an ohm. On the other hand, an insulating material may possess a resistance of several millions of ohms, referred to as *megohms* (M Ω), one megohm being a million ohms. The international standard ohm is taken as the resistance at 0° C. (32° F.) of a column of mercury 106·3 cm. long and having a uniform sectional area of 1 sq. mm.

The mercury column is, of course, only used as a standard of reference, and in practice where a known resistance is required for measurement or regulating purposes a special alloy wire is wound on a bobbin or former. These resistances, or *resistors* as they are called, are procurable in all values from a fraction of an ohm up to one megohm. If a resistor is desired to be continuously variable a single

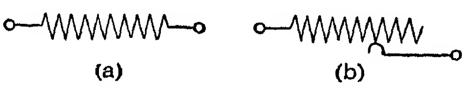

FIG. 5.—Fixed and variable resistors.

layer of bare wire is wound on a long cylinder and a movable contact is made to rub along the wire. In Fig. 5 is shown the conventional manner in which fixed and variable resistors are represented in a diagram.

Resistance and Dimensions of a Conductor.

If two wires of the same material but having different dimensions are compared they will be found to possess

different resistances. Firstly, taking two wires of the same sectional area, or gauge, but one of the wires having twice the length of the other, it will be found that the longer wire offers twice the resistance of the shorter wire.

In general, let l represent the length of the wire, then $R \propto l$, i.e. the resistance is directly proportional to the length.

Secondly, taking two wires of the same length, but one having twice the sectional area of the other, it will be found that the larger wire offers one-half of the resistance of the smaller wire.

In general, let a represent the sectional area of the wire, then $R \propto \frac{1}{a}$, i.e. the resistance is *inversely* proportional to the sectional area.

Resistance and Material of a Conductor.

It is found that different materials, although of the same length and sectional area, possess different resistances. In other words, the nature of the material, as well as its dimensions, affects the resistance. For example, an iron wire of given dimensions has roughly 8 times the resistance of a similar copper wire, while an alloy of copper and nickel called " eureka " has some thirty times the resistance of the copper wire. Hence, before it is possible to calculate the resistance of a material of known dimensions it is necessary to have a knowledge, found experimentally, of its resistance referred to stated dimensions. These dimensions may be either a length of 1 in. and sectional area of 1 sq. in. or a length of 1 cm. and sectional area of 1 sq. cm. The resistance for these standard dimensions is termed the *resistivity* (or sometimes the *specific resistance*) and is denoted by the symbol ρ (pronounced " rho "). Thus ρ is a constant for any given material and the expression for the resistance of a conductor now becomes:

$$R = \rho \times \frac{l}{a}.$$

If ρ is given in inch units, then l must be in inches and a in square inches and if ρ is given in centimetre units, l must be in centimetres, and a in square centimetres. The

resistivity of copper at average room temperature is 0·0000007 ohm-in. or 0·0000017 ohm-cm. To obviate the use of the noughts, a smaller unit called the *microhm* ($\mu\Omega$) is employed, one microhm being one millionth part of an ohm.

The above values for resistivity of copper then become 0·7 microhm-in. and 1·7 microhms-cm. respectively.

Example. To find the resistance of 100 yards of copper wire having a sectional area of 0·00102 sq. in. (No. 20 S.W.G.)

$$R=\frac{0{\cdot}0000007\times(100\times36)}{0{\cdot}00102}$$

$$=2{\cdot}47 \text{ ohms.}$$

Ohm's Law.

It has been seen that the total resistance of a circuit is given by dividing the e.m.f. by the current. It therefore follows that the current passing in a circuit is obtained by dividing the e.m.f. by the resistance, or,

$$I=\frac{E}{R}.$$

In this form, the expression is usually referred to as Ohm's Law. Transposing the simple equation above, we may also write :

$$E=I\times R.$$

Expressed in words, this means that the e.m.f. or total pressure acting in a circuit is given by the product of the current and the total resistance of the circuit.

Internal Resistance of a Cell.

Referring to the simple circuit of Fig. 4, a reading is taken of the voltmeter with the switch S open, i.e. with no current flowing. The switch is then closed and it will be found that the reading of the voltmeter is now less than the original reading. Thus the pressure existing at the terminals of the cell is less when delivering current than on open circuit. This pressure is termed the pressure difference or *potential difference* (p.d.) to distinguish it from the e.m.f. which is the total pressure exerted by the cell, the latter being given by the voltmeter reading on open

circuit. The p.d. is less than the e.m.f. because some pressure is required to overcome the internal resistance of the cell.

The current read on the ammeter in Fig. 4 is not given by $\frac{E}{R}$ but by $\frac{\text{p.d.}}{R}$, where R is the resistance of the lamp; it is, however, given by $\frac{E}{R+r}$ where r is the internal resistance of the cell and $(R+r)$ is therefore the total resistance of the circuit (neglecting the resistances of the ammeter and connecting wires). When applying Ohm's Law to find the current it is the external p.d. across a known resistance which must be employed. If the e.m.f. is used then this must be divided by the sum of the external

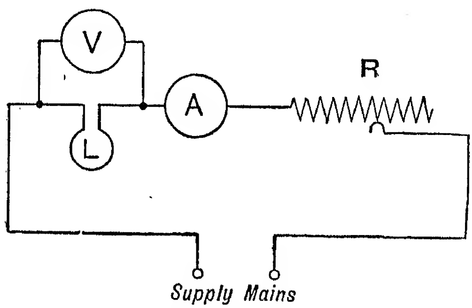

FIG. 6.—Resistance of lamp filament.

and internal resistances. The pressure lost in overcoming the internal resistance of the cell is given by $I \times r$, so that if V represents the terminal p.d.,

$$V = E - I \times r.$$

The following table of readings taken when testing a Leclanché cell should make clear the above explanations.

E	I	V	r	R	$\frac{V}{R}$	$\frac{E}{R+r}$
1·5	0·5	1·4	0·2	2·8	0·5	0·5

It will be seen that the current, I, read on the ammeter is given by both $\frac{V}{R}$ and $\frac{E}{R+r}$.

The important point to remember is that Ohm's Law applies to any part of an electric circuit as well as to the whole circuit. If the p.d. across any portion and the resistance of that portion are known, then the current may be calculated from $\frac{V}{R}$.

Also, if the p.d. and current relating to any portion are measured by a voltmeter and ammeter respectively, then the resistance of that portion may be calculated from $\frac{V}{I}$.

Again, if the current passing through a known resistance is measured, the p.d. across that resistance is given by $I \times R$.

What has been said about the p.d. across the terminals of a cell being less than the e.m.f. of the cell applies equally to a generator or, indeed, any source of e.m.f. Where, therefore, it is of importance that the pressure required to operate a particular piece of apparatus should be of a certain value it must be remembered that the e.m.f. will require to be greater than the desired p.d. in order to allow for the internal voltage drop.

Effect of Temperature on Resistance.

It has long been known that the temperature of a conductor affects its resistance and this can best be demonstrated by conducting a test on an incandescent lamp where the temperature may be varied over very wide limits. Fig. 6 shows a lamp L connected in series with an ammeter and variable resistor R to supply mains. The voltage across the lamp is read on the voltmeter and this may be varied by means of R from a low value up to the rated voltage of the lamp. The resistance of the lamp filament may be calculated from the voltmeter and ammeter readings and if these readings are plotted in the form of a graph as in Fig. 7 it is seen that the curve is not a straight line which indicates that the current is not proportional to the p.d. and, therefore, the resistance of the lamp filament is

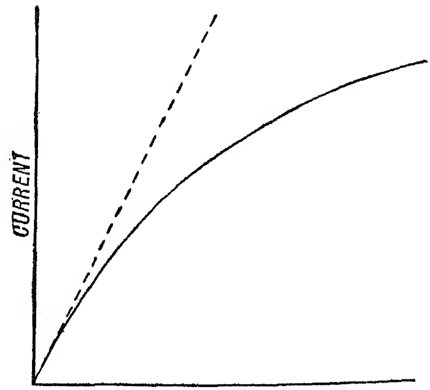

FIG. 7.—Voltage-current for lamp filament.

varying. The dotted line graph shows how the cu would vary if the resistance did not change. In I the resistance of the lamp filament has been plotted ag the current and it will be seen that this increases current, i.e. with increasing temperature of the fila

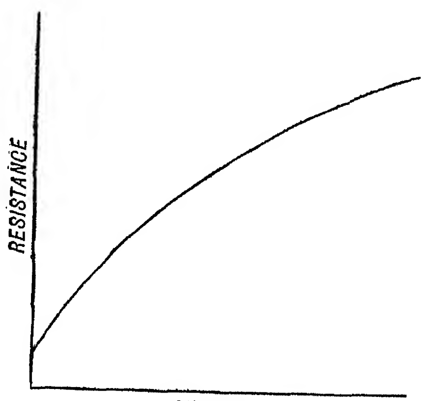

FIG. 8.—Variation of resistance of lamp.

The filament of an incandescent electric lamp is made of tungsten and it is found that other pure metals, such as copper, aluminium, iron, etc., also increase in resistance with increase of temperature. Carbon, on the other hand, which is a fair conductor, decreases in resistance as the temperature is increased.

Certain alloys, however, such as eureka, constantan and manganin, which are alloys of copper and nickel, show no appreciable change in resistance with variation of temperature and are therefore employed in the construction of standard resistances as used for measurement purposes and for extending the range of voltmeters and ammeters.

Temperature Coefficient.

The increase (or decrease) in resistance of a material having a resistance of 1 ohm at 0° C. when raised in temperature by 1° C. is termed the *temperature coefficient of resistance*, and is usually denoted by the symbol α (pronounced "alpha"). Thus, in the case of copper, a length of wire having a resistance of 1 ohm at 0° C. offers a resistance of 1·0043 ohms at 1° C., so that the temperature coefficient for copper is 0·0043. At 20° C. (average room temperature) the copper wire would have a resistance of $1+(20\times0{\cdot}0043)=1+0{\cdot}086=1{\cdot}086$ ohms. A coil of copper wire having a resistance of 100 ohms at 0° C. would have a resistance at 20° C. of $100+(100\times0{\cdot}0043\times20)=100+8{\cdot}6=108{\cdot}6$ ohms. This represents an increase of 0·43 ohm per degree rise in temperature or 0·43 per cent. In general, if R_0 be the resistance at 0° C. then the *increase* of resistance at t° C. is $R_0\alpha t$.

If R be the resistance at t° C., then:

$$R=R_0+R_0\alpha t=R_0(1+\alpha t).$$

In practice, the resistance at 0° C. is not known, but the resistance at some particular temperature, usually room temperature, may be measured and it is desired to know what the resistance would be at some higher temperature. Alternatively, the resistance at the higher temperature may be also measured and it is required to know the rise in temperature.

Let R_1 and R_2 be the resistances corresponding to t_1° C. and t_2° C. respectively, then:

$$R_1 = R_0(1+\alpha t_1)$$

and $$R_2 = R_0(1+\alpha t_2)$$

hence $$\frac{R_1}{R_2} = \frac{1+\alpha t_1}{1+\alpha t_2}$$

or $$R_2 = R_1\{1+\alpha(t_2-t_1)\} \text{ very nearly.}$$

Example 1. A solenoid of copper wire has a measured resistance of 120 ohms at room temperature of 20° C. After the solenoid has been in circuit for three hours, the resistance is again measured and is found to be 150 ohms. It is desired to know the increased temperature of the coil.

$$150 = 120\{1+0{\cdot}0043(t_2-t_1)\}$$

or $$1{\cdot}25 = 1+0{\cdot}0043(t_2-t_1)$$

and $$t_2-t_1 = \frac{0{\cdot}25}{0{\cdot}0043} = 58^\circ \text{ C.}$$

Since $t_1 = 20^\circ$ C., the new temperature is 78° C.

Example 2. A coil of copper wire of 60 ohms at room temperature of 16° C. is raised to 66° C. It is required to know the new resistance.

$$R_2 = 60(1+0{\cdot}0043\times 50)$$
$$= 60(1{\cdot}215) = 72{\cdot}9 \text{ ohms.}$$

Alternatively, since the increase in resistance is 0·43 per cent. per ° C. rise in temperature, total increase in resistance is $0{\cdot}43\times 50$ or 21·5 per cent. This is $\frac{21{\cdot}5}{100}\times 60$ or 12·9 ohms, and new resistance is $60+12{\cdot}9$ or 72·9 ohms, as before.

Resistors in Series.

In Fig. 9 is shown three resistors, R_1, R_2 and R_3 connected in series to a source of supply of V volts. Since there are no alternative paths for the current to take it is clear that the value of the current must be the same anywhere in the circuit. This can be shown by connecting the ammeter at various points in the circuit and its reading will remain unchanged. The readings of the three voltmeters V_1, V_2 and V_3 are next noted and these may show different values,

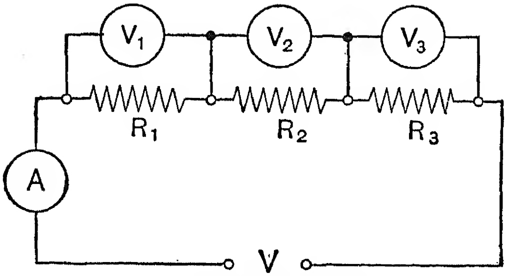

FIG. 9.—Resistors in series.

but the sum of the readings will add up to the supply voltage V.

Hence, $V = V_1 + V_2 + V_3$
$= IR_1 + IR_2 + IR_3$

Let R = total resistance of whole circuit
then, $V = IR$
and $IR = IR_1 + IR_2 + IR_3$
or $R = R_1 + R_2 + R_3$.

In words, the total resistance of a number of resistors in series is given by the sum of the resistances of the individual resistors.

Example 1. In a four-valve radio receiver for d.c. mains the heaters of the valves are connected in series. Each heater requires a p.d. of 16 volts and a current of 0·25 ampere. The mains voltage being 230, it is desired to know what value of resistor must be placed in series with the heaters.

Each heater has a resistance of $\frac{16}{0{\cdot}25}$ or 64 ohms, so that the four in series will have a resistance of 4×64 or 256 ohms. But the total resistance of the circuit must be $\frac{230}{0{\cdot}25}$ or 920 ohms. Hence the resistor to be connected in series must have a value of $920 - 256$ or 664 ohms.

An alternative method of calculation is as follows:

Voltage required for the four heaters in series is 4×16

Example 1. Three coils A, B and C have resistances 2, 4 and 8 ohms respectively. It is required to know the equivalent resistance when they are connected in parallel.

Total conductance,
$$\frac{1}{R}=\frac{1}{2}+\frac{1}{4}+\frac{1}{8}$$
$$=0{\cdot}5+0{\cdot}25+0{\cdot}125$$
$$=0{\cdot}875 \text{ mho.}$$

Equivalent resistance,
$$R=\frac{1}{0{\cdot}875}=1{\cdot}14 \text{ ohms.}$$

Example 2. A moving coil instrument has a resistance of 100 ohms and requires a current of 1 milliampere to produce full-scale deflection. It is desired to extend the range of this instrument so that it will read up to 100 milliamperes.

As the instrument itself must only pass 1 milliampere with 100 milliamperes flowing in the circuit it is clear that a small resistance must be placed in parallel as shown in Fig. 11. This resistance, or *shunt*, must carry 99 milli-

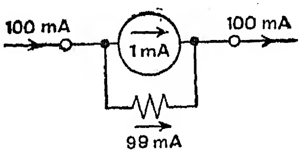

FIG. 11.—Extension of range of milliammeter.

amperes. The p.d. across the instrument is given by 0.001×100 or $0{\cdot}1$ volt and this p.d. is also across the shunt. Hence the resistance of the latter is $\frac{0{\cdot}1}{0{\cdot}099}$ or $1{\cdot}01$ ohms. It will be noted that the resistance of the instrument, when used on the 1 milliampere range is 100 ohms, but the combined resistance of instrument and shunt when used on the 100 milliampere range is $\frac{0{\cdot}1}{0{\cdot}1}$ or 1 ohm.

The Potential Divider.

It is sometimes necessary in practice to be able to vary the p.d. applied to a piece of apparatus over wide limits, from zero to the maximum available voltage of supply. A variable resistor in series is a possible method and may be used where the current passing is comparatively large. It will be realised, however, that in order to reduce the p.d. across the apparatus to a very low value a high value of series resistor becomes necessary. A much better method, where the current to be taken by the apparatus is small, is to use a variable resistor as a *potential divider.* This method is shown in Fig. 12 where terminals A and B

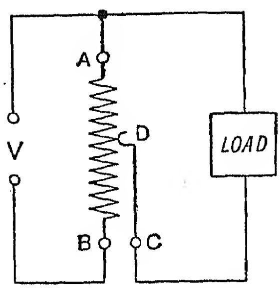

FIG. 12.—The potential divider.

represent the two ends of the resistor and are connected to the supply voltage V. The local circuit, marked "load," is then connected to terminal A and terminal C, the latter being attached to the bar along which the sliding contact D moves. The p.d. applied to the load is determined by the voltage drop across the portion AD of the resistor and as D may be moved from A to B the p.d. to the load may be varied from zero to a value V. As the whole of the resistor is permanently connected across the supply voltage its resistance must be so chosen that the current taken is not excessive for the physical size of the wire.

Electrical Power and Energy.

It has been seen in Chapter I that when a current passes through a resistance heat is generated. As heat is a form

Commercial Unit of Electrical Energy.

Although the unit of energy has been given as the joule, a larger unit is used for the buying and selling of electrical energy. This unit is the *kilowatt-hour* (kWh) and is often referred to as the *Board of Trade Unit* because it is the legal unit on which charges for energy are based.

Example. A radiator rated at 2 kilowatts is in use, on the average, for 5 hours a day and it is required to know the weekly consumption of energy and also the cost, if the price of energy is $\frac{5}{8}$ penny per kilowatt-hour.

Weekly consumption $=2\times5\times7=70$kWh.

Cost $=70\times\frac{5}{8}=43\frac{3}{4}$ pence or $3/7\frac{3}{4}$.

CHAPTER IV

ELECTROMAGNETISM

The Magnetic Field.

A bar magnet when pivoted or suspended so as to be free to move in a horizontal plane will take up a position pointing roughly north and south, due to the magnetic influence of the Earth. The end of the magnet which points to the north is called the *north-seeking pole* and the other end the *south-seeking pole* or briefly, *north* (N.) and *south* (S.) *poles* respectively.

If now the N. pole of another magnet is brought near the N. pole of the pivoted magnet there is a force of repulsion, but if the S. pole of the second magnet is brought near the N. pole of the pivoted magnet a force of attraction results.

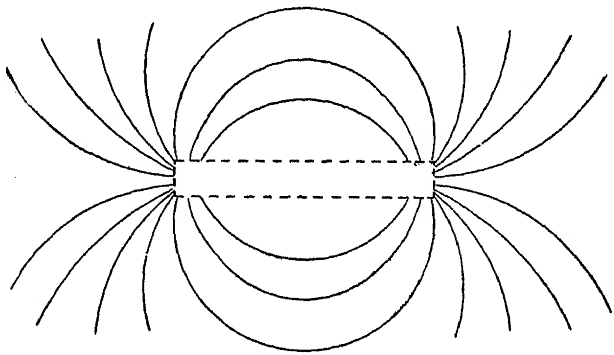

FIG. 13.—Distribution of magnetic field due to bar magnet.

A second experiment consists in placing a sheet of cardboard over the magnet and to sprinkle iron filings on the sheet when it will be seen that the filings set themselves in curved lines between the poles of the magnet, as shown in Fig. 13. The shape and density of these lines is very

helpful in forming a mental picture of the magnetic conditions existing in the neighbourhood or *field* of the magnet and the lines are spoken of as *magnetic lines of force*, the strength of the field at any point being measured by the density in lines per square centimetre.

Every line of force is imagined to be a closed loop with the return path through the steel of the magnet and they may be likened to stretched elastic threads, so that if the magnetic force is destroyed the lines will immediately collapse.

Direction of a Magnetic Field.

The direction of a magnetic field is taken as that direction in which the N. pole of a small pivoted magnet points when placed in the field. Thus, in Fig. 14 is a bar magnet placed on a flat surface and a number of small compass needles

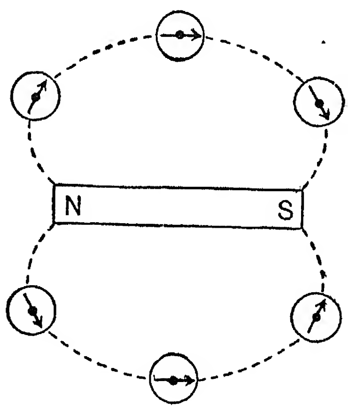

FIG. 14.—Direction of magnetic field.

placed in the positions shown, when it will be found that they set themselves along the lines indicated in Fig. 13. The arrow heads on the needles represent the N. poles, so it is said that the lines of force leave at the N. pole and enter at the S. pole. The strength of the magnet may be expressed by the number of lines of force leaving the N.

pole (or entering the S. pole) and this is spoken of as the *magnetic flux.* The crowding of the lines of force in the field will be greatest in the vicinity of the poles and will become less and less as the distance from the magnet increases, so that the term *flux density* is used to denote the strength of the field at any point. Flux density is the number of lines of force passing through an area of 1 sq. cm., drawn at right angles to the lines. A bar magnet, for example, might produce a magnetic flux of 600 lines and if the sectional area of the magnet is 1·5 sq. cm. the flux density adjacent to each pole, and in the body of the magnet itself, would be $\frac{600}{1 \cdot 5}$ or 400 lines per sq. cm. The flux density a few inches away from the magnet would be very much less.

Magnetic Field of a Straight Conductor.

In Fig. 15 a length of stout copper wire W is shown passing at right angles through a sheet of cardboard. If a current of fairly high value is passed through the wire and iron filings are sprinkled on the cardboard, it will be

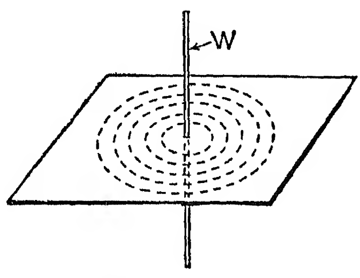

FIG. 15.—Field of a straight conductor.

seen that the filings set themselves into concentric circles with the axis of the wire as the common centre. It is clear that the current is producing a magnetic effect in the neighbourhood of the conductor. The direction of the lines of force may be determined by placing compass needles on

the cardboard sheet and noting the direction in which the N. poles point. This is shown in Fig. 16 for a current passing down the wire, indicated by a cross, and for a current passing up the wire, indicated by a dot. The direction of the lines of force depends, therefore, on the direction of the current and the rule to remember the

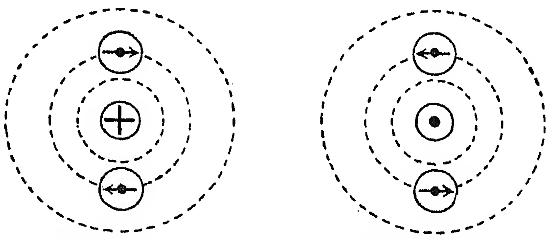

Fig. 16.—Direction of field due to straight conductor.

direction is the screw rule. Imagine a right-handed screw being driven in the direction of the current, then the rotary motion of the screw (and hand) gives the direction of the lines of force.

As would be expected, the flux density is greatest near the surface of the wire and falls off rapidly as the distance from the wire increases.

Magnetic Field of a Coil.

The field produced by a straight wire is comparatively weak, but if the wire is wound to form a coil the magnetic effects produced by each unit length of wire are then concentrated so that all the lines of force pass through the inside of the coil. In Fig. 17 is shown a single layer coil in section and the direction and distribution of the lines of force. It will be seen that the coil behaves like a bar magnet and exhibits N. and S. poles at the two ends of the coil. If the direction of the current is reversed, the polarity of the coil will also reverse. If, looking endways at a coil the direction of the current is clockwise, that end

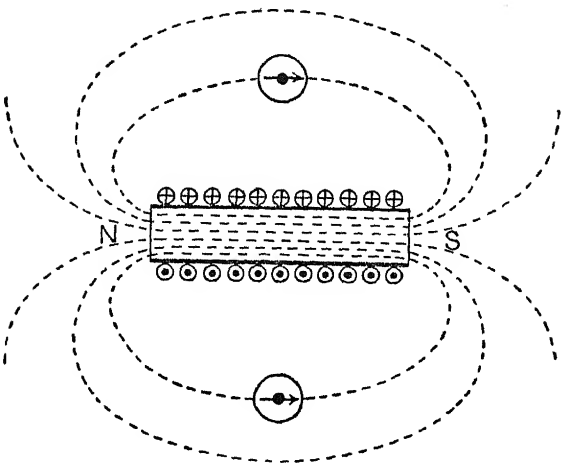

Fig. 17.—Magnetic field of a coil.

is the S. pole. This rule may be remembered by writing the letter S and placing arrows at each end as shown in Fig. 18. The arrows indicate the direction of the current around the coil and the S denotes the polarity of the end of the coil being observed.

It will be realised that since all the flux passes through the inside of the coil, the strength of the field there, as measured by the flux density, is at its greatest and is weak outside the coil.

The strength of the field inside for a given length of coil is proportional to two factors :

(1) The current.
(2) Number of turns.

i.e. to the product of current and turns, or the *ampere-turns*.

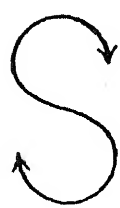

Fig. 18.—Rule for polarity of a coil.

Thus, a coil of many turns and carrying a small value of current will produce the same magnetic effect as another coil of few turns but carrying a large value of current, provided the ampere-turns are the same for both coils.

Iron-cored Coil.

It has been assumed so far that the core (the material inside the coil) has been air ; the same value of flux would be obtained if the core had been of wood, brass, aluminium, or indeed almost any substance with the exceptions of iron and steel. If, however, a bar of iron is inserted inside the coil the flux is enormously increased and the iron becomes a powerful magnet so long as the current is passing through the turns of the coil. The iron, therefore, has the ability to increase the flux, compared with air, etc., for a given number of ampere-turns. Various grades of iron and steel produce varying values of flux ; a good soft wrought iron

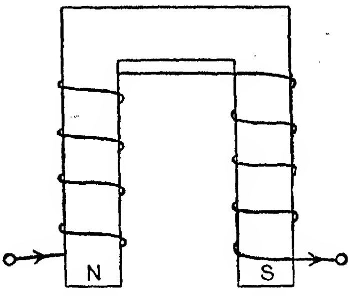

FIG. 19.—Electromagnet.

gives the greatest increase in flux whilst cast iron gives the lowest. Steel causes a fair increase and, moreover, retains a considerable proportion of the flux even when the current has been switched off. Soft iron, on the other hand, loses practically all its magnetism when the current is stopped, which explains why it is used for the magnet cores in electric bells, relays, etc. In Fig. 19 is shown a typical electromagnet such as might be found in an electric bell. It will be noticed that the poles are brought close together so that the path of the lines of force in air is reduced as far as possible.

A *permanent magnet* is produced by subjecting a steel alloy to the influence of a strong magnetic field for a short space of time. Modern permanent magnets are usually

de of "alnico" steel, the trade name of an alloy .sisting of aluminium, nickel, cobalt and iron. Permanent gnets are used in magnetos, moving coil instruments, ·phone receivers, etc.

ɜ Magnetic Circuit.

n Fig. 20 is shown a coil (in two portions) wound on a ıed iron core so that there are no definite poles. The ın path taken by the lines of force is shown by the ted line. Such a magnetic circuit is found in chokes

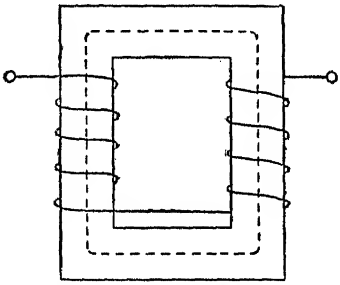

FIG. 20.—A closed magnetic circuit.

transformers. If the iron is not present and assuming mean path of the flux is unchanged the strength of field in lines per sq. cm. is given by $1{\cdot}257 \times \frac{IT}{l}$, where the current passing through the coil in amperes, T is total number of turns in the coil and l is the length ıe mean path of the flux in cms. This shows that the is proportional to the ampere-turns per centimetre of th.

now, the measurement is repeated with the iron core osition it is found that the flux density is increased he same value of ampere-turns. The ratio of the flux ity in the iron to the strength of the field (also measured ıes per sq. cm.) is termed the *permeability* of the iron. ıe strength of the field (magnetising force) is denoted ., the flux density in the iron by B and the permeability (pronounced "mu"), so that $\mu = \frac{B}{H}$.

If a graph is plotted connecting B and H the shape is as shown in Fig. 21 from which it is seen that the curve is not a straight line, which means that the ratio of B to H varies according to the value of H.

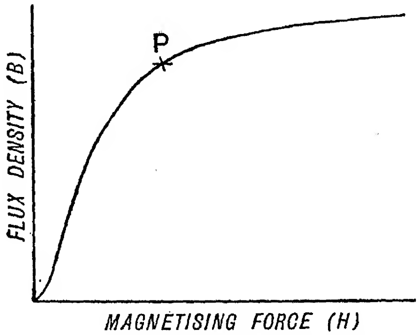

FIG. 21.—Magnetisation curve for iron.

It will also be noted from the graph that when the flux density has reached a value corresponding to the point marked P a large increase in the value of H produces only a slight increase in the flux density. This point is termed the *saturation point* of the iron and the value of the flux density at which this occurs is of the order of 14,000 lines per sq. cm. for a soft wrought iron and 7,000 lines per sq. cm. for cast iron. Corresponding values of the permeability are 1000 for the soft iron and 200 for the cast iron. Let a represent the sectional area of the iron in sq. cm. then the total lines of force or flux (denoted by Φ) is given by $\mathrm{B}\times a$ so that

$$\begin{aligned}\Phi &= \mathrm{B}a\\ &= \mu \mathrm{H}\times a\\ &= \mu\times\frac{1{\cdot}257\mathrm{IT}}{l}\times a\\ &= \frac{1{\cdot}257\mathrm{IT}}{\frac{1}{\mu}\times\frac{l}{a}}\end{aligned}$$

1·257IT represents the *magnetomotive force* (m.m.f.) of the circuit and $\frac{1}{\mu} \times \frac{l}{a}$ is termed the *reluctance*.

Hence, in words, $\text{flux} = \frac{\text{m.m.f.}}{\text{reluctance}}$.

This expression is very similar to that for the electric circuit, viz.:

$$\text{current} = \frac{\text{e.m.f.}}{\text{resistance}}.$$

Example. Considering the magnetic circuit of Fig. 20, the sectional area of the soft iron core is 20 sq. cm. and the mean length of path of the flux is 25 cm. It is desired to know the ampere-turns required to produce a flux of 280,000 lines in the core.

$$\text{Flux density (B)} = \frac{280{,}000}{20} = 14{,}000 \text{ lines per sq. cm.}$$

Referring to the B–H curve for soft iron (supplied by the manufacturer) it is found that the strength of the field (H) required is 14 units. The permeability (μ) is $\frac{14{,}000}{14}$ or 1000, and the reluctance of the iron is $\frac{1}{1000} \times \frac{25}{20}$ or 0·00125 unit.

$$\text{Hence, } 280{,}000 = \frac{1{\cdot}257\text{IT}}{0{\cdot}00125}$$

$$\text{and} \qquad \text{IT} = \frac{280{,}000 \times 0{\cdot}00125}{1{\cdot}257} = 280 \text{ ampere-turns.}$$

This means that if 1 ampere is to pass through the turns of the coil, 280 turns will be required.

Magnetic Circuit with Air-Gap.

In many magnetic circuits in practice there is an air-gap which, however, is kept as short as possible because of the high reluctance of air compared with iron. The necessity for the air-gap arises due to the existence of a moving element which must be free to deflect or rotate. Examples of the former are the moving coil instrument and the

armature of the electric bell and relay and the chief example of the latter is the armature of the electric motor and generator. Air-gaps are also sometimes found in the iron cores of chokes and the reason here is that it is desired to keep the flux density, and hence the flux, sensibly proportional to the current through the winding so that a graph connecting flux and current would be as shown in Fig. 22 instead of that as shown in Fig. 21 for a closed

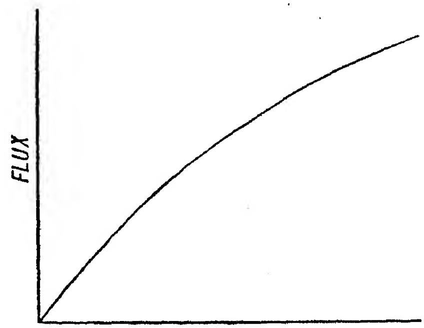

FIG. 22.—Magnetisation curve for core with air-gap.

iron core. It must be appreciated, however, that the presence of the air-gap increases the number of ampere-turns required to produce a given value of flux and this is illustrated in the following example.

Example. Referring to Fig. 23, the circuit shown is identical with that of Fig. 20 except that an air-gap of 0·5 cm. exists in the iron core and it is desired to know what will be the necessary number of ampere-turns to produce the same flux as in the previous example.

The reluctance of the iron path to the flux is now $\frac{1}{1000}\times\frac{24{\cdot}5}{20}$ or 0·0012 unit.

The reluctance of the air-gap is $\frac{1}{1}\times\frac{0{\cdot}5}{20}$ or 0·025 unit.

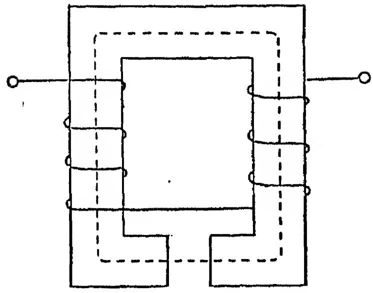

FIG. 23.—Magnetic circuit with air-gap.

(The permeability of air and all non-magnetic materials is 1·0.)

Total reluctance$=0{\cdot}0012+0{\cdot}025=0{\cdot}0262$ unit.

Hence $$280{,}000=\frac{1{\cdot}257IT}{0{\cdot}0262}$$

and $$IT=\frac{280{,}000\times 0{\cdot}0262}{1{\cdot}257}=5{,}830 \text{ ampere-turns.}$$

It will be seen that the presence of the air-gap has necessitated an increase of over 20 times in the number of ampere-turns required to produce the same flux.

Hysteresis.

Referring again to Fig. 20, suppose the current through the winding is varied in steps from zero to some maximum value, then decreased back to zero, after which the p.d. is reversed and the current (now flowing in the opposite direction) increased to the same maximum value as before and back again to zero. Finally, the direction of p.d. is restored to the original direction and the current increased once more to the maximum value. As the current is varied, the flux, and flux density, is also being varied and the iron is said to have been taken through a *cycle of magnetisation*. This occurs in the cores of chokes, transformers and the armatures of generators and motors. In the case of chokes and transformers the current through the windings will be alternating and therefore the flux will be varying from instant to instant. In the case of armatures the iron

is subjected to the magnetic effect of north and south p
alternately as rotation takes place.

To illustrate the effect of taking the closed iron through a cycle of magnetisation, curves will be plo connecting flux density (B) and magnetising force (H) this is shown in Fig. 24.

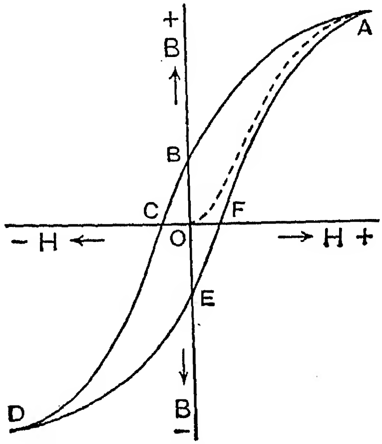

FIG. 24.—Hysteresis loop.

Starting with an unmagnetised iron core the dotted l curve OA is obtained and this is the B–H curve of Fig. : As the current (and hence H) is decreased it is found tl the flux follows the line AB until with H at zero some fl remains as is shown by OB. This residual flux densi is termed the *remanence* of the iron or steel. Continui with the cycle, it is seen that a reversed magnetising foı (—H) of a value represented by OC is required to destr the remanence. OC is termed the *coercive force*. Proceedi to the same maximum value for —H as for +H the cur takes the path CD and, for decreasing values, back fro D to E. OE is again the remanence but the iron magnetised in the reverse direction. Reverting to +] OF is now the coercive force and the cycle is final completed along the path FA. The complete figu

ABCDEFA is called the *hysteresis loop* for the particular specimen of iron under test and the area enclosed by the loop is a measure of the energy loss due to hysteresis. This loss appears as heat in the iron core and is additional to the I^2R loss in the winding.

In general, the harder the steel the greater will be the hysteresis loss so that in the cases mentioned where the iron is being continuously subjected to cycles of magnetisation a special soft grade of iron must be employed in order to minimise the hysteresis loss.

If a non-magnetic material is subjected to a cycle of magnetisation the resulting graph is a straight line through the origin showing that no loss takes place due to hysteresis.

Conductor carrying Current in a Magnetic Field.

In Fig. 25 is shown a conductor carrying current away from the reader and situated in a magnetic field with its length at right angles to the lines of force. The current

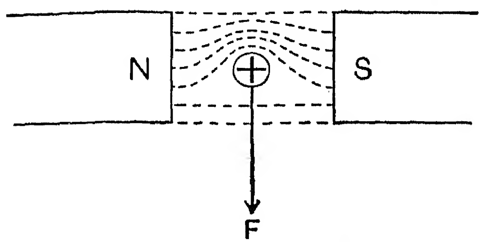

FIG. 25.—Conductor in a magnetic field.

tends itself to set up concentric lines of force in a clockwise direction with the effect that the resultant field is crowded above the conductor and less dense below. Since the lines of force want to take the shortest path from the N. to S. poles the conductor experiences a mechanical force F in the direction shown by the arrow and movement of the conductor may result. If either the direction of the field or the current be reversed the direction of the force is also reversed. A convenient rule to obtain the direction of the force is illustrated in Fig. 26. Point the first finger of the *left hand* in the direction of the lines of force and the second

finger in the direction of the current, when the thumb extended will give the direction of the force exerted on the conductor tending to move it perpendicularly to the axis of the conductor and to the direction of the magnetic field.

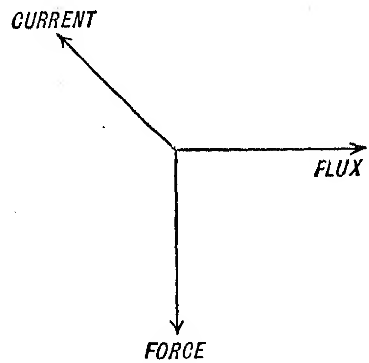

FIG. 26.—Force on a conductor.

The force on the conductor is given by $\frac{BlI}{9810}$ grammes, where B is the density of the field in lines per sq. cm., l is the length of the conductor actually situated in the magnetic field in cms., and I is the current in the conductor in amperes. Applications of the above effect are found in moving coil instruments and in electric motors.

Example. The armature of an electric motor contains 100 conductors, each 20 cm. long and carrying a current of 50 amperes. The conductors are situated in a magnetic field of 9000 lines per sq. cm. and it is desired to find the total force acting on the conductors in pounds weight.

$$\text{Force per conductor} = \frac{9000 \times 20 \times 50}{9810}$$

$$= 917 \text{ grammes.}$$

Total force $= 917 \times 100 = 91{,}700$ grammes.
But 453·6 grammes $= 1$ pound,

and total force $= \frac{91{,}700}{453 \cdot 6} = 200$ pounds approx.

CHAPTER V

ELECTROMAGNETIC INDUCTION

E.M.F. Induced in a Coil.

In Fig. 27 is shown a coil C wound on a hollow former and connected to a galvanometer, i.e. a sensitive moving coil instrument with its zero in the centre of the scale. A magnet is brought up from a distance and inserted inside

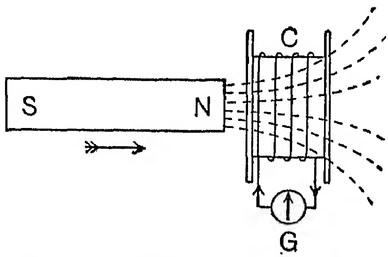

FIG. 27.—E.M.F. induced in a coil.

the coil so that the lines of force thread or link with the turns of the coil. If the pointer of the galvanometer is observed it will be noticed that there is a momentary deflection, say to the right, when the magnet is near the coil *and in motion.* As soon as the magnet is at rest the pointer of the galvanometer returns to zero. If the magnet is now removed from inside the coil there is again a momentary deflection, but to the left, showing that the direction of the current in the coil has reversed. If the S. pole of the magnet is brought up to the coil the galvanometer pointer will deflect to the left. Identical effects to the above are obtained if the coil is brought up or removed from the magnet showing that a *relative motion* between the magnet and the coil causes an e.m.f. to be produced (or *induced*) in the latter. The phenomenon of electromagnetic induction may now be stated in the following form :

Whenever there is a *change* in the number of mag lines of force linked with the turns of a coil an e.m induced in the coil.

Magnitude of Induced E.M.F.

Referring again to Fig. 27, if the turns on the coil increased in number it will be found that the current hence the e.m.f.) is proportionately increased. Furth the time taken to bring up (or remove) the magn shortened the e.m.f. is again increased, i.e. the ind e.m.f. increases as the *rate of linking* with the turns incre Finally, if a stronger magnet is used the number of of force linked with the coil will be greater and the ind e.m.f. is again increased. If Φ is the change in the nun of lines of force linked with the coil, T the number of t in the coil and t the time in seconds for the linking to place, then the average value of the induced e.m.f. is g by $E=\frac{\Phi T}{10^8 t}$ volts. (10^8=100,000,000, and is emplo here to convert the e.m.f. from scientific units to v the practical unit.)

Direction of Induced E.M.F.

It is clear that if the induced e.m.f. is allowed to s a current by completing the external circuit to the electrical energy is being produced and, in conseque mechanical work will have to be performed on the mov system, i.e. either the magnet or the coil. From consideration the law, known as *Lenz's law*, follo *The direction of the induced e.m.f. (and current) is such to oppose the motion producing it.*

Thus, in Fig. 27, the end of the coil facing the N. p of the magnet will act as a N. pole and tend to repel approaching magnet. The current will therefore flow a counterclockwise direction looking at the left-hand e of the coil.

E.M.F. Induced in a Conductor.

A conductor may be looked upon as part of a one-tu coil and it is sometimes convenient to consider the induc

e.m.f. in a single conductor. This is particularly useful in the case of the armature conductors of generators. In Fig. 28 is shown a conductor AB falling through a magnetic field so as to cut the lines of force, the resulting momentary

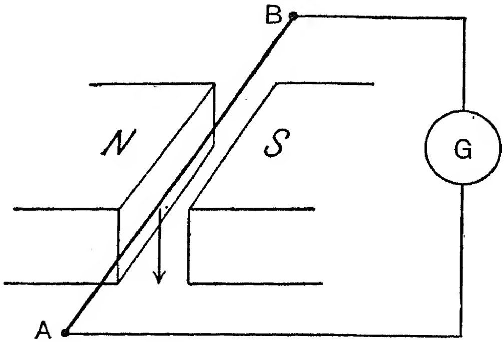

FIG. 28.—E.M.F. induced in a conductor.

current causing a deflection of the galvanometer G indicating that an e.m.f. has been produced. The direction of the current through AB will be such as to set up a mechanical force opposing the downward motion of the conductor. Employing the left-hand rule, illustrated in Fig. 26, it is found that the current will flow from B to A. Alternatively, and this is more convenient, Fleming's *right-hand rule* may be employed. Here, the first finger of the right hand is pointed in the direction of the flux, as in Fig. 29, and the thumb in the direction of motion of the conductor, then the second finger represents the direction of the induced e.m.f.

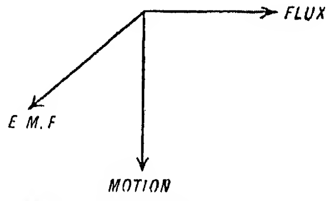

FIG. 29.—Right-hand rule.

Applying this rule to Fig. 28, it is seen that the direction of e.m.f. (and current) is from B to A, as obtained previously. If l is the length of the conductor in the

magnetic field in cm. and d is the distance moved through the field in cm., then the area (A) swept out by the conductor at right angles to the lines of force is ld sq. cm. But $\Phi=BA$, where B is the density of the field in lines per sq. cm., and for a one-turn coil, $E=\frac{\Phi}{10^8 t}$. Substituting for Φ, it is seen that:

$$E=\frac{BA}{10^8 t}=\frac{Bld}{10^8 t}.$$

But $\frac{d}{t}$ is the speed or velocity (v) of the conductor in cm. per second.

Hence, $$E=\frac{Blv}{10^8} \text{ volts.}$$

The Simple Generator.

Electromagnetic induction has an important application in the generator as used in power stations for the supply of electrical energy. By means of rotation a continual change takes place in the magnetic flux linking with the turns of a coil, and by so doing an e.m.f. is constantly being generated. An electromagnet supplies the flux amounting to several millions of lines of force per pole and this is referred to as the *field-system*. The winding, consisting of a large number of turns in series, with which the flux links is called the *armature*. Either the field system or armature may rotate. In d.c. generators it is the armature which is made to rotate, but in a.c. generators (alternators) the armature is stationary and the field system rotates. In Fig. 30 is illustrated the principle of the generator, where is shown a coil of one turn (two conductors) rotating about the axis XY in a magnetic field. The two ends of the coil are connected to two "slip rings" SS mounted on, but insulated from, the shaft. Two "brushes" (of carbon) BB pressing on the slip-rings provide contact between the rotating coil and the external circuit R. In the position shown in the figure, the *change* of flux with the coil is momentarily zero and hence the e.m.f. generated is also zero. Applying the alternative view of induction by the cutting of flux by a conductor, it is seen

that the two conductors AB and CD are momentarily moving parallel to the lines of force and hence no cutting is taking place showing, as before, that the e.m.f. generated is zero. It is now proposed to follow the coil through a complete revolution and this is illustrated in Fig. 31. In position (*a*) the e.m.f. (as already seen) is zero. A quarter of a revolution later is shown at (*b*) and in this position the e.m.f. is momentarily at its maximum value since the conductors AB and CD are cutting the flux at right angles.

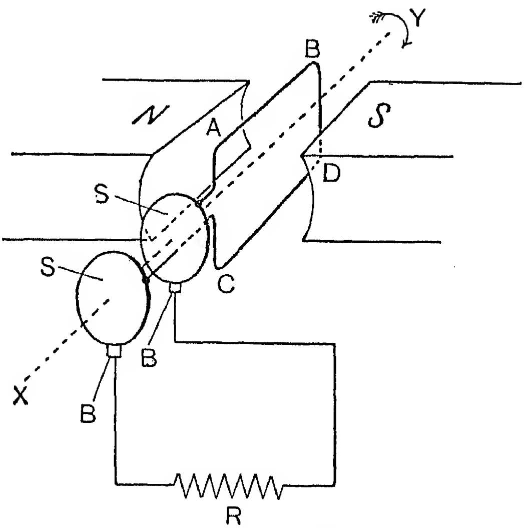

FIG. 30.—Principle of the generator.

After half a revolution (shown at (*c*)) the e.m.f. is again zero while in position (*d*) the e.m.f. is once more at its maximum value but the direction of the induced e.m.f. is reversed compared with position (*b*). This is readily seen by employing the right-hand rule and it will be found that taking conductor AB the e.m.f. in position (*b*) is towards the reader, whereas in position (*d*) the direction of the e.m.f. is away from the reader. The current through the external circuit R of Fig. 30 will therefore be alternating and each

revolution of the coil will give rise to one complete *cycle* of current. If the coil is made to rotate at 3000 revolutions per minute there will be generated 50 complete cycles in one second, since the speed of the coil is 50 revolutions per second.

The variation of the e.m.f. in the coil from instant to instant during a revolution is shown graphically in Fig. 32. 0° corresponds to position (*a*) in Fig. 31, 90° to position (*b*), 180° to position (*c*) and 270° to position (*d*). Between

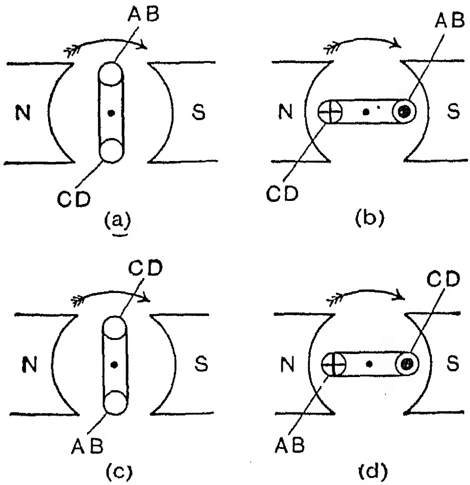

FIG. 31.—Rotation of coil in magnetic field.

180° and 360° the direction of the induced e.m.f. is reversed and this is shown in the graph by plotting below the base line and placing a minus sign on the vertical line of e.m.f.

The more or less flat tops to the two half cycles is due to the poles being curved, thus keeping the air-gap between conductors and poles uniform, and to the coil being wound on a soft iron cylinder to reduce the reluctance of the magnetic circuit to the lines of force. The flux under the poles therefore enters and leaves the iron cylinder radially and the conductors cut the lines of force at right angles,

thus maintaining a fairly constant e.m.f. while passing under the poles.

Example 1. A conductor 15 cm. long moves through a magnetic field of density 8000 lines per sq. cm. at right angles to the flux with a velocity of 25 metres per second. Calculate the average value of the induced e.m.f. in the conductor.

Using the formula $E=\frac{Blv}{10^8}$,

$$E=\frac{8000\times15\times2500}{10^8}=3 \text{ volts.}$$

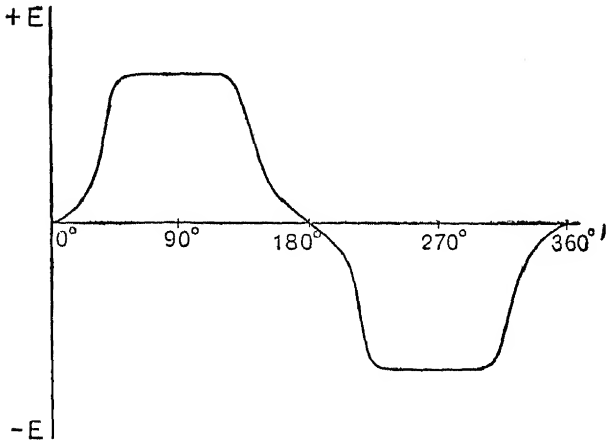

FIG. 32.—E.M.F. in rotating coil.

Example 2. A coil of 100 turns is rotated in a two-pole magnetic field at a speed of 3000 revolutions per minute. The flux is 2 megalines (2,000,000 lines). Calculate the average value of the e.m.f. induced in the coil.

All the flux per pole is linked with the coil in one quarter of a revolution, e.g. in going from position (*b*) to position (*c*) in Fig. 31.

The time taken for the coil to move through one quarte of a revolution is $\frac{60}{4\times 3000}$ or $\frac{1}{200}$ second.

Using the expression $E=\frac{\Phi T}{10^8 t}$,

$$E=\frac{2\times 10^6\times 100}{10^8\times\frac{1}{200}}=\frac{2\times 10^6\times 100\times 200}{10^8}=400 \text{ volts.}$$

The Direct Current Generator.

If it is desired to render the p.d. and current in the external circuit unidirectional, a rectifier or *commutator* may be fitted on the shaft of the rotating armature. For a

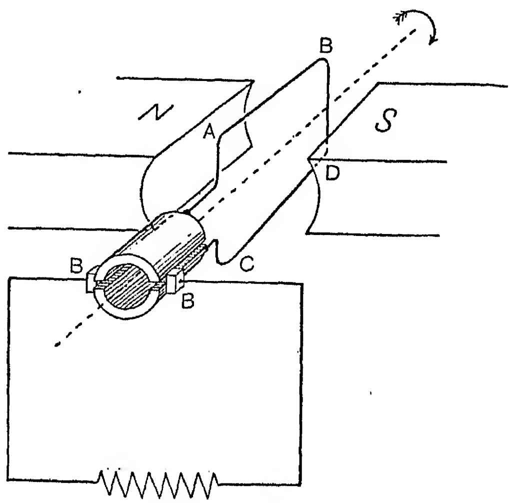

FIG. 33.—Coil and commutator.

single coil armature, the commutator consists of two copper segments insulated from each other by mica sheets and from the shaft by a ring of micanite. The ends of the coil are connected to the two segments and brushes press

against the commutator. The arrangement is shown diagrammatically in Fig. 33 and this should be compared with Fig. 30. The position of the brushes is important and

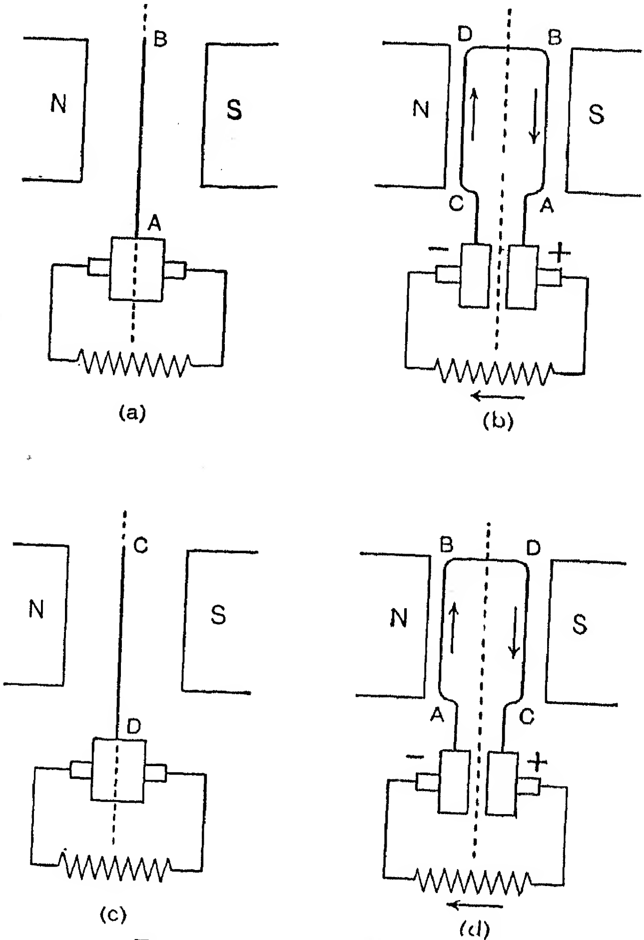

FIG. 34.—Commutator action.

these should be so placed as to press on the mica sheets when the conductors AB and CD are cutting no flux. The sequence of events as the coil is rotated through one revolution will now be followed from Fig. 34. In position (*a*) the e.m.f. is zero, in position (*b*) e.m.f. is a maximum and with direction of e.m.f. and current as shown i.e. with the right-hand brush of positive polarity. In position (*c*) the e.m.f. is again zero and in position (*d*) it is seen that the right-hand brush is still of positive polarity so that the current *in the external circuit* flows in the same direction as in position (*b*). The alternating current in the conductors AB and CD has therefore been rectified by the commutator and a unidirectional current obtained in the external circuit.

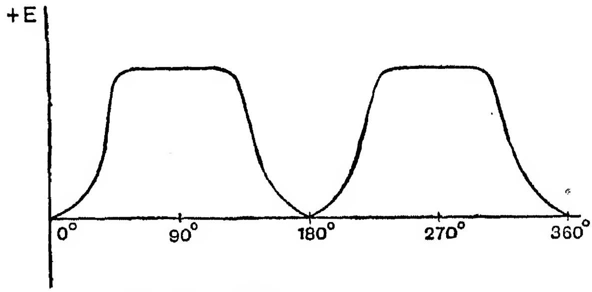

FIG. 35.—E.M.F. with commutator.

The explanation is simple; in position (*b*) the conductor AB is in contact with the positive brush but in position (*d*), when the current in AB has reversed, the conductor is in contact with the negative brush. A graph of the e.m.f. obtained at the brushes is shown in Fig. 35, from which it will be seen that this is pulsating. In order to render the e.m.f. and current less fluctuating a number of coils connected in series, and distributed around the surface of the iron core of the armature, are employed, and the commutator is built up of the same number of copper segments as armature coils. The junction of the end of one coil and the beginning of the next is connected to each commutator segment.

The Direct Current Motor.

The construction of a machine to be used as a motor from a d.c. supply is identical with that of a generator. The commutator is still required so that as a conductor on the armature moves from under the N. pole to under the S. pole the current in that conductor has reversed in direction and in consequence the mechanical force is always acting in the same direction. Coils placed on the poles and connected in series are supplied with direct current and create the magnetic field. These are referred to as the field coils or winding. In the case of alternators a separate source of d.c. supply is required to feed the field winding but in the case of d.c. machines the source is available at the brushes.

Field Excitation.

If the field winding is in parallel with the armature the machine is referred to as a "shunt" motor (or generator) and the connections are shown in the diagram of Fig. 36, where F represents the field winding and A the armature.

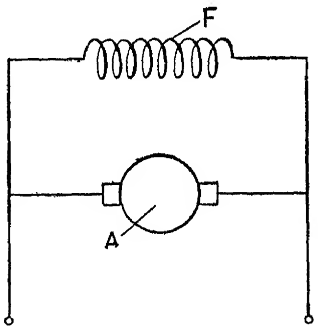

FIG. 36.—Shunt machine.

Since the field winding is directly across the voltage of supply the coils will be wound with a large number of turns of small diameter wire and the field current will be low—of the order of 1 or 2 amperes. A variable resistor in series with the field winding enables the field current to be varied

and hence the e.m.f. to be controlled in the case of a generator and the speed in the case of a motor.

If the field winding is connected in series with the armature, this is referred to as a "series" machine and

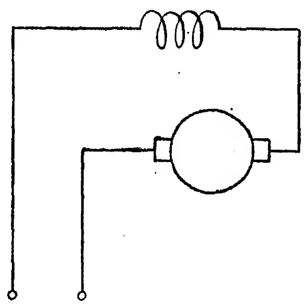

FIG. 37.—Series machine.

this method of excitation is shown in Fig. 37. The current passing through the armature also passes through the field winding and as this will be comparatively large when the

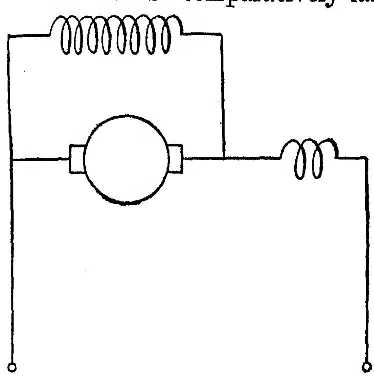

FIG. 38.—Compound machine.

machine is on load the coils will be wound with a few turns of thick wire.

In some machines the field excitation consists of two windings, one connected in shunt and the other in series.

Such a machine is referred to as a "compound" machine and is illustrated diagrammatically in Fig. 38. The directions of the currents through the two windings will be such that they both tend to set up flux in the same direction.

Starting a Motor.

The resistance of the armature of a machine will be a fraction of an ohm, except in small sizes. If, therefore, the full voltage of supply is applied to the armature when at rest an excessive current will flow and it becomes necessary to apply the voltage gradually, allowing time between each increase of voltage for the armature to gather speed. As the resistance of the armature cannot alter it becomes necessary to enquire how it is that the full voltage can be applied when the armature is running at normal speed without an excessive current flowing. When the armature commences to rotate, an e.m.f. is induced in the conductors and by Lenz's law this e.m.f. tends to oppose the applied voltage so that the current through the armature is given by $I_a=\frac{V-E}{R_a}$, where V is the applied voltage, E is the induced e.m.f. and R_a is the resistance of the armature including brushes.

Since the e.m.f. generated in the armature of a motor opposes the applied voltage it is often referred to as the *back e.m.f.*

Example. The voltage at the brushes of a d.c. machine is 230 volts and the resistance of its armature 0·2 ohm; if the armature carries a current of 20 amperes, find the e.m.f. generated (*a*) when the machine is running as a generator and (*b*) when running as a motor.

Voltage drop in armature$=20\times0{\cdot}2=4$ volts.

(*a*) $$E=V+I_aR_a$$
$$=230+4=234 \text{ volts.}$$

(*b*) $$E=V-I_aR_a$$
$$=230-4=226 \text{ volts.}$$

Fig. 39 shows the connections of a shunt motor to the starting resistor, R, known as a *starter*. The latter is a

variable resistance, usually in the form of an arm moving over studs connected to resistance coils. On closing the switch all the resistance R is in series with the armature but the full voltage of supply is across the field winding. As the armature speeds up, the resistance R may be cut out step by step until finally the supply voltage is connected directly across the brushes.

A compound motor is started in a similar manner but

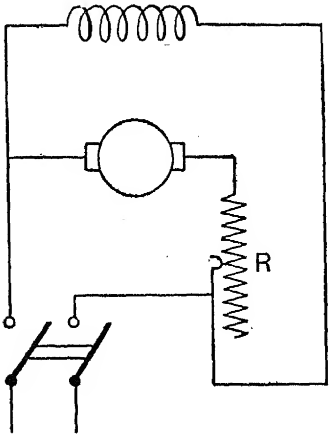

FIG. 39.—Starting a shunt motor.

in the case of the series motor the starting resistor is in series with both armature and field.

A variable resistor in series with the shunt field winding is often met with in d.c. machines; in the case of a generator this is used as a means of varying the e.m.f. and in a motor for the control of speed. In the latter case, a decrease of field current causes an *increase* in the speed since a smaller field current means less flux and, in consequence, the armature will rotate at a higher speed in order to generate the same value of back e.m.f.

CHAPTER VI

INDUCTANCE

Self Induction.

Consider the circuit of Fig. 40 in which is shown a coil connected to a battery through an ammeter and switch S. If the coil is wound with a large number of turns on an iron core, then on closing the switch it is found that the

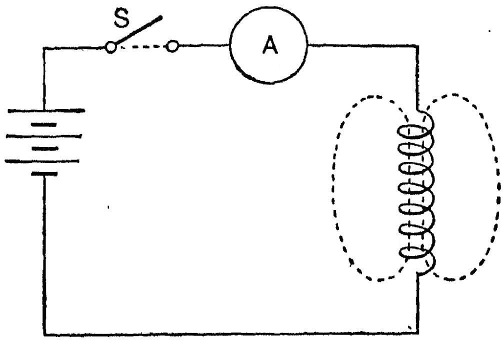

FIG. 40.—Self induction.

current read on A takes an appreciable time (a matter of a few seconds) to reach a steady value. If the experiment is repeated, using the element of an electric heater in place of the coil, it is found that the current rises instantly to its final value. The explanation of the difference in the two cases is found in the strong magnetic field set up by the coil, whereas the heater element produces practically no flux. An increase in the current causes a change in the number of lines of force linked with the coil—shown dotted in Fig. 40—and this, in turn, induces an e.m.f. in the coil which, by Lenz's law, tends to oppose that producing it, in this case the growth of current. The induced e.m.f. is, therefore, a back e.m.f. and acts in opposition to the applied voltage. When the current has reached a steady state there is no further change in the lines of force

and the back e.m.f. becomes zero. If, now, the switch is opened, the current dies away and again there is a change in the flux linked with the coil and an e.m.f. is induced, tending to maintain the current. As the conducting path for the current is broken at the switch considerable arcing may take place at the gap between the switch contacts. The induced e.m.f. in this case is in the same direction as the applied voltage.

Any circuit in which a change of current produces a change in the flux linked with that circuit, and therefore is accompanied by an induced e.m.f., is said to be *inductive* or to possess *self-inductance* or simply *inductance*.

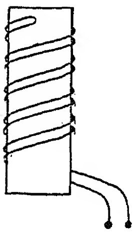

FIG. 41.—Non-inductive resistance coil.

A circuit in which but little flux is set up by a current is termed a *non-inductive* circuit, and for most practical purposes such circuits as lamp and valve filaments, heater elements, etc., may be regarded as non-inductive. In the case of precision resistance boxes used for measurement purposes it is very desirable that the inductance should be reduced to a minimum, and the wire is doubled back on itself before being wound on an insulating former, as shown in Fig. 41. In this method of winding the magnetising force produced by one conductor is neutralised by the adjacent conductor, so that little or no flux links with the turns of the coil.

Unit of Inductance.

The inductance (denoted by L) of a coil or circuit is measured in units called *henrys* (denoted by H), and is said to have an inductance of 1 henry if the induced e.m.f. is 1 volt when the current is changing at the rate of 1 ampere per second. If the current changes from I_1 to I_2 amperes in t seconds and E is the induced e.m.f., then:

$$L = \frac{E}{\frac{I_2 - I_1}{}} \text{ henrys}$$

and $$E = L \times \frac{I_2 - I_1}{t} \text{ volts.}$$

Example 1. The e.m.f. induced in a coil is 10 volts when the current through it is changed from 1 to 5 amperes in 0·1 second. Calculate the inductance.

$$\text{Rate of change of current}=\frac{5-1}{0\cdot 1}=40 \text{ amperes per second.}$$

$$L=\frac{10}{40}=0\cdot 25 \text{ H.}$$

Example 2. The field winding of a machine has an inductance of 20 henrys and carries a current of 1 ampere. Calculate the induced e.m.f. if the circuit is broken in 0·01 second.

Rate of change of current $=\frac{1}{0\cdot 01}=100$ amperes per second.

$$E=20\times 100=2000 \text{ V.}$$

This high value of e.m.f. induced is liable to damage the insulation of the winding so that highly inductive circuits must be broken slowly, or alternatively must be shunted by a resistance before the current is interrupted.

Alternative Expression for Inductance.

Suppose a current of I amperes through a coil of T turns produces a flux of Φ lines and let the current grow from 0 to I amperes in t seconds. The rate of change of current is $\frac{I}{t}$ amperes per second and the change of flux linked with the coil is Φ lines.

From above, $L=\frac{E}{\frac{I}{t}}$, but as seen previously, $E=\frac{\Phi T}{10^8 t}$

so that:

$$L=\frac{\frac{\Phi T}{10^8 t}}{\frac{I}{t}}=\frac{\Phi T}{10^8 I} \text{ henrys.}$$

The product ΦT is termed the "flux-linkages." A coil, therefore, possesses an inductance of 1 henry when a change of 1 ampere produces a change of 100,000,000 flux-linkages.

Example. It is found that a coil of 100 turns produces a flux of 100,000 lines when a current of 5 amperes is passing. Calculate the inductance of the coil.

$$L=\frac{100{,}000\times 100}{10^8\times 5}=0{\cdot}02 \text{ henry.}$$

Calculation of Inductance.

If the number of turns on a coil is doubled the flux will be doubled for a given current, but the number of flux-linkages will be four times as great, so that it follows the inductance will be increased four-fold. Hence the inductance is proportional to the square of the number of turns. The expression derived in Chapter IV for the flux produced in a magnetic circuit was seen to be: $\Phi=\dfrac{1{\cdot}257IT}{\frac{1}{\mu}\times\frac{l}{a}}$.

Now $L=\dfrac{\Phi T}{10^8 I}$ so that substituting for Φ, we get:

$$L=\frac{1{\cdot}257IT\times T}{\frac{1}{\mu}\times\frac{l}{a}\times 10^8 I}=\frac{1{\cdot}257}{10^8}\times\frac{\mu a T^2}{l} \text{ henrys.}$$

Hence the inductance is also proportional to the sectional area and inversely proportional to the length of the magnetic circuit.

If the coil is wound on a non-magnetic core the permeability is 1·0, and for small inductances as met with in the high frequency stages of radio receivers and transmitters this is common practice. It is then more convenient to express the inductance in microhenrys (μH), one microhenry being one-millionth part of a henry. The expression for air-cored coils then becomes: $L=0{\cdot}01257\times\dfrac{aT^2}{l}$ microhenrys. It is only possible to use this expression for the calculation of air-cored inductances if the coil is close wound on a ring, when l becomes the mean circumference of the ring. For short, straight coils, if l in the formula is taken as the length of the coil a correction factor must be employed.

Iron-cored Inductances.

For larger inductances, such as are met with in the low-frequency stages of radio receivers and transmitters, and as smoothing "chokes" in rectifiers, the coil is wound on a closed soft-iron core. The permeability of the iron may be as high as 10,000, which means that, compared with a non-magnetic core of the same dimensions, the inductance is 10,000 times as great. It must be remembered, however, that the flux produced in an iron core is not proportional to the current through the windings, so

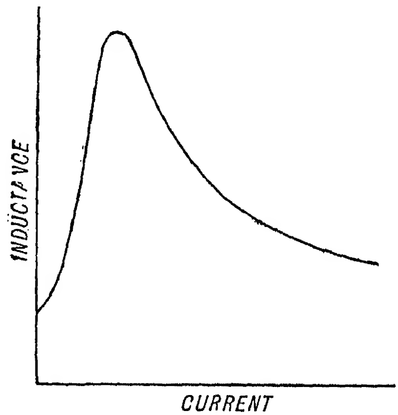

FIG. 42.—Inductance of iron-cored coil.

that the permeability, and hence the inductance, will vary with the current. Fig. 42 is a graph showing how the inductance of an iron-cored coil will vary with the current. It is necessary, therefore, when quoting the inductance, to specify the current through the coil. If it is desired that the inductance should be more or less independent of the current, an air-gap is inserted in the iron core which has the effect of causing the flux to increase proportionally with the current.

Example 1. Fig. 43 shows an iron-cored inductance wound with 1000 turns. The mean length of a line of

force (shown dotted) is 25 cm. and the sectional area of the core (as at AB) is 10 sq. cm. Calculate the inductance when 100 milliamperes are passing through the winding.

The magnetising force (H) is given by,

$$1{\cdot}257 \times \frac{IT}{l} = 1{\cdot}257 \times \frac{0{\cdot}1 \times 1000}{25} = 5 \text{ units.}$$

Referring to the magnetisation curve for the particular iron, it is found that the flux density (B) corresponding to a magnetising force of 5 units is 10,000 lines per sq. cm.

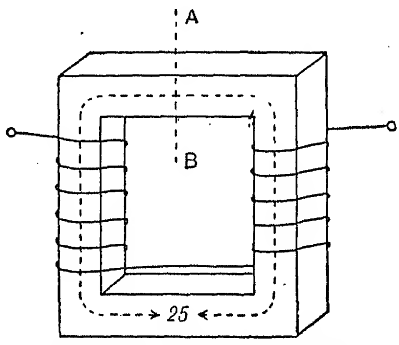

FIG. 43.—Inductance example.

$$\text{Permeability } (\mu) = \frac{B}{H} = \frac{10{,}000}{5} = 2000.$$

$$\text{Inductance} = \frac{1{\cdot}257}{10^8} \times \frac{2000 \times 10 \times 1000^2}{25} = 10 \text{ henrys.}$$

An alternative solution is as follows:

$$\Phi = Ba = 10{,}000 \times 10 = 100{,}000 \text{ lines}$$

$$L = \frac{\Phi T}{10^8 I} = \frac{100{,}000 \times 1000}{10^8 \times 0{\cdot}1} = 10 \text{ henrys.}$$

Example 2. Using the same winding and dimensions as in the previous example, but with a non-magnetic core, calculate the inductance.

$$\text{Inductance}=0{\cdot}01257\times\frac{10\times1000^2}{25}=5000 \text{ microhenrys}$$

or 0·005 henry.

This result is obvious, for with a non-magnetic core the permeability is unity and therefore the inductance with the iron core will be 2000 times greater than the inductance with the air core.

Growth of Current in an Inductive Circuit.

It has been noted that if a battery is connected to an inductive circuit the current does not rise immediately to the final value, due to the back e.m.f. set up by the inductance when the current is changing.

If the circuit possessed no resistance, the back e.m.f. would be constant in magnitude and equal to the applied voltage V of the battery.

But, back e.m.f. $E=L\times$rate of change of current, so that, rate of change of current$=\frac{E}{L}=\frac{V}{L}$ amperes per sec. This means that the graph of current against time would be a straight line and the current would grow indefinitely. Actually, the final value ofthe current is, of course, determined by Ohm's law, viz. $\frac{V}{R}$, so that if the current grew at a constant rate of $\frac{V}{L}$ amperes per second the final value of current would be reached in $\frac{L}{R}$ seconds.

This period of time is referred to as the *time constant* of the circuit. A circuit to illustrate the growth of current is shown in Fig. 44, where, for convenience, the inductance and resistance have been separated and connected in series.

In Fig. 45 is depicted a graph showing the growth of current in an inductive circuit. OB represents the slope of the curve at the instant of switching on, when the rate of change is $\frac{V}{L}$ amperes per second. OA is the final value

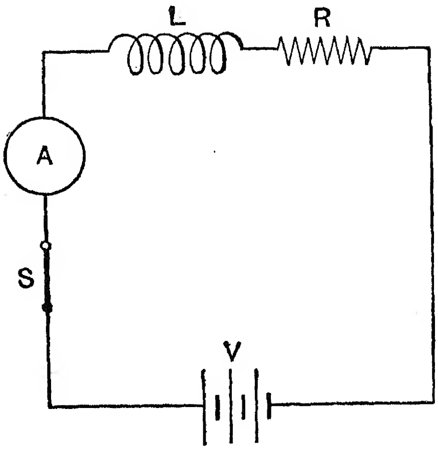

FIG. 44.—An inductive circuit.

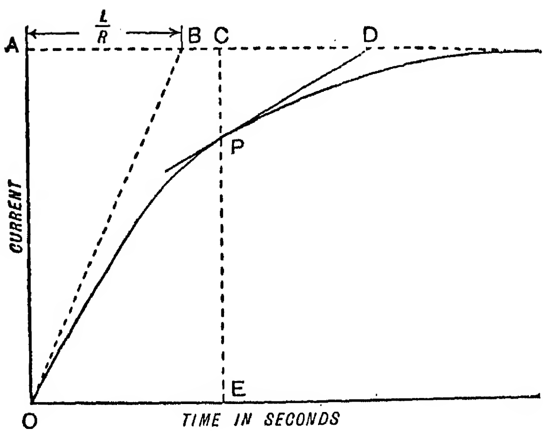

FIG. 45.—Growth of current in inductive circuit.

of the current $\left(\frac{V}{R}\text{ amperes}\right)$, so that AB in seconds is the time constant of the circuit. Take any point P on the curve and draw a tangent PD. Then $\frac{PC}{CD}$ gives the rate of change of current at the point P. The instantaneous value of the current at the point P is PE and this may be represented by i. $PC=\frac{V}{R}-i$, since CE is the final value of the current, viz. $\frac{V}{R}$.

Let e represent the instantaneous value of the back e.m.f. at point P, then:

$$V=e+iR$$
$$=L\times\frac{PC}{CD}+iR$$

$$\text{or } V-iR=L\times\frac{PC}{CD}=L\times\frac{\frac{V}{R}-i}{CD}$$
$$=\frac{L}{R}\times\frac{V-iR}{CD}$$
$$\text{hence } CD=\frac{L}{R}.$$

This means that at any point on the curve, as well as at the start, the rate of growth of current is such that if it continued at the same rate it would reach the final value of current in $\frac{L}{R}$ seconds.

Actually, it can be shown that in $\frac{L}{R}$ seconds from the instant of switching on, the current has reached 63 per cent. of its final value.

Energy Stored in a Magnetic Field.

When a magnetic field is created energy is absorbed but no further energy is required to maintain the field,

Energy is therefore stored in the magnetic field and when this is allowed to collapse, as on breaking a circuit, the whole of the energy stored is released and usually appears as heat losses in the circuit. Suppose the current in a coil, having an inductance of L henrys, grows at a uniform rate from zero to I amperes in t seconds, as depicted in Fig. 46. The average value of the current will be $\frac{1}{2}$I amperes and the e.m.f. induced in the coil $L \times \frac{I}{t}$ volts. A portion of the applied voltage, equal to $\frac{LI}{t}$, will be

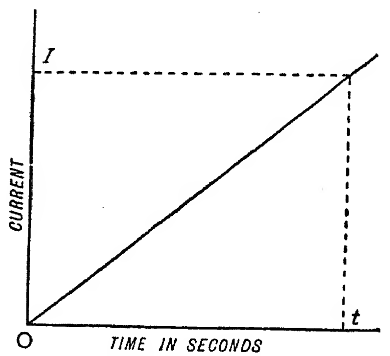

FIG. 46.—Energy stored in a magnetic field.

required to overcome this back e.m.f. Hence average power absorbed by the magnetic field

$$=\frac{LI}{t}\times\tfrac{1}{2}I$$

$$=\tfrac{1}{2}\times\frac{LI^2}{t}\text{ watts}$$

and energy absorbed by the magnetic field

$$=\tfrac{1}{2}\times\frac{LI^2}{t}\times t$$

$$=\tfrac{1}{2}LI^2\text{ watt-seconds or joules.}$$

Mutual Induction.

Consider two coils, A and B, placed close to each other as in Fig. 47. Coil A is connected to a battery and switch S and coil B to a galvanometer with a centre zero scale. On closing the switch, a current flows in coil A and lines of force are set up, some of which will link with the turns of coil B, as shown by dotted lines in the figure. As a result of the change in flux-linkages an e.m.f. will be induced in coil B and there will be a momentary deflection on the galvanometer, say to the right of zero. On opening

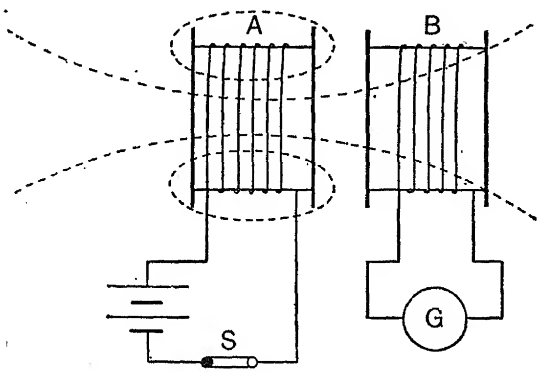

FIG. 47.—Mutual induction.

the switch, the current in A, and also the lines of force will collapse and again there will be an e.m.f. induced in B and the galvanometer needle will be momentarily deflected to the left of zero. This effect of producing an e.m.f. in a second coil by means of a changing current in the first coil is known as *mutual induction* and the two coils are said to possess *mutual inductance.*

Mutual inductance (denoted by M) is measured in henrys, and two coils have a mutual inductance of 1 henry when an e.m.f. of 1 volt is induced in the second coil due to the current in the first coil changing at the rate of 1 ampere per second. If the current in the first coil changes from

I_1 to I_2 amperes in t seconds and E_2 is the e.m.f. induced in the second coil :

$$M=\frac{E_2}{\frac{I_2-I_1}{t}} \text{ henrys}$$

$$\text{and} \quad E_2=M\times\frac{I_2-I_1}{t} \text{ volts.}$$

Alternative Expression for Mutual Inductance.

Let the current in the first coil (known as the *primary*) change from zero to I_1 amperes in t seconds and let the flux *actually linking with the turns of the second coil* (known as the *secondary*) be Φ lines of force. Then, if the secondary has T_2 turns, the e.m.f. induced, $E_2=\frac{\Phi T_2}{10^8 t}$ volts.

$$\text{hence} \quad M=\frac{E}{\frac{I_1}{t}}$$

$$=\frac{\frac{\Phi T_2}{10^8 t}}{\frac{I_1}{t}}$$

$$=\frac{\Phi T_2}{10^8 I_1} \text{ henrys.}$$

By increasing the number of turns T_2 in the secondary it will be realised that any desired value of e.m.f. may be obtained. Mutual inductance between two coils represents the principle of the alternating current transformer and of the ignition or spark coil. In all cases, except for very high frequencies, the two coils are wound on a common iron core, so that practically all the flux produced by the primary links with the secondary.

CHAPTER VII

CAPACITANCE

Electrical Condenser or Reservoir.

An atom of any substance, under normal conditions, contains equal quantities of positive and negative electricity, and the body is said to be in an uncharged state. It is, however, possible to remove, or add to, the negative charges within the material and this is readily accomplished in the case of metals. These very small negative charges are caled *electr ons* and hold a quantity of electricity equal to $\frac{1{\cdot}58}{10^{19}}$ of a coulomb. A metal body which has less than its normal number of electrons is said to be positively charged and one which has a surplus is said to be negatively charged. If a conducting wire is connected between a positively charged body and a negatively charged body, electrons will flow from the latter to the former until each body has its normal number. By the old convention, established before the discovery of the electron, the direction of flow is from the positively charged body (i.e. higher potential) to the negatively charged body (i.e. lower potential), and this convention is still employed. Electrons in motion constitute a current, and 1 ampere represents a rate of flow of $6{\cdot}28 \times 10^{18}$ electrons per second. In Fig. 48 is shown two metal plates, A, B, separated by an insulator and connected through a centre-zero ammeter and switch S to a battery. On closing the switch, electrons will be attracted away from plate A by the positive pole of the battery and will therefore become positively charged, while the same number of electrons will be added to plate B which will therefore become negatively charged. During this transference of electrons there will be a momentary deflection of the ammeter needle. The switch may now be opened, and it will be found that a potential difference exists between the plates A and B, of the same value as the voltage of the battery. The two plates, separated by an insulator, constitute what is termed an *electrical con-*

denser and it is an arrangement which acts as a reservoir for storing a quantity of electricity.

If, now, the battery in Fig. 48 is removed and replaced

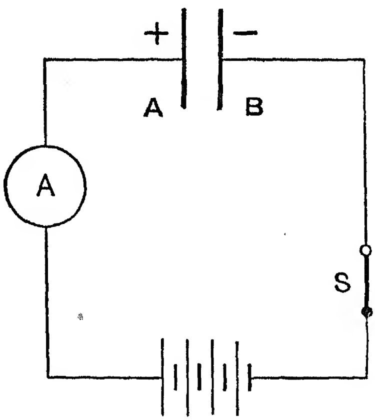

FIG. 48.—Electrical condenser.

by a wire, then on closing the switch there will be a deflection of the ammeter needle in the opposite direction to that obtained previously.

Charge and Discharge Curves.

When the battery is connected to the condenser the initial current will be determined by $\frac{E}{R}$ where E is the e.m.f. of the battery and R the resistance of the circuit (not including, of course, the very high resistance of the insulator between the plates). As the condenser becomes charged a p.d. is set up across the plates which opposes the voltage of the battery, so that the current falls and becomes zero when the p.d. is equal to the e.m.f. of the battery. The condenser is now said to be charged. On discharging the condenser, the initial value of current is the same as on charge, but in the opposite direction, and falls away in a similar manner to zero as the p.d. across

the plates decreases to zero. Curves for charge and discharge currents are shown in Fig. 49, where the discharge is depicted as taking place immediately after the condenser has been charged. The areas under the two curves

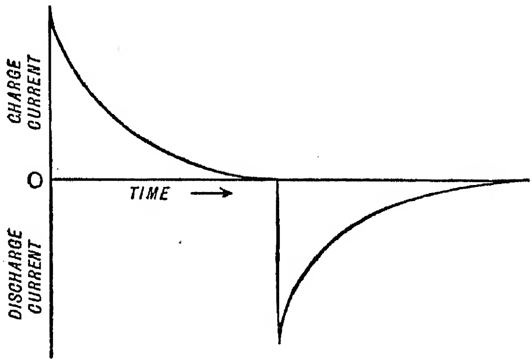

Fig. 49.—Charge and discharge curves.

are identical and represent the quantities of electricity flowing during charge and discharge. When a condenser has been charged it will retain that charge indefinitely so long as there is no leakage through the insulator between the plates and until such time as it is discharged.

Capacitance.

It is found that the quantity in coulombs which a condenser will store is directly proportional to the charging voltage. The electrical size or *capacitance* of a condenser is defined as the quantity stored (in coulombs) when a p.d. of 1 volt is applied. If, on applying a p.d. of 1 volt to a condenser, 1 coulomb of electricity is stored, the condenser is said to have a capacitance of 1 *farad* (symbol F). In general, if C is the capacitance in farads, Q is the quantity stored in coulombs and V is the applied voltage,

$$\text{then } C=\frac{Q}{V}$$

$$\text{and } Q=CV.$$

In practice, the farad is an inconveniently large unit and the *microfarad* (denoted by μF) is used, the microfarad being one-millionth part of a farad. In radio work, a still smaller unit is often met with, the *micromicrofarad* ($\mu\mu$F), one micromicrofarad being one-millionth part of a microfarad.

Thus, 500 $\mu\mu F = 0{\cdot}0005\ \mu F$.

Example. A condenser with a capacitance of 50 microfarads is connected to a d.c. supply of 600 volts. Calculate the quantity of electricity stored.

$$Q = C\ (\text{farad}) \times V\ (\text{volts})$$
$$= \frac{50}{10^6} \times 600$$
$$= 0{\cdot}03\ \text{coulomb.}$$

Condensers in Parallel.

In Fig. 50 is shown two condensers, C_1 and C_2, connected in parallel across a battery of voltage V. The voltage across each condenser is the same, namely V.

Quantity, Q_1, stored in $C_1 = C_1V$
Quantity, Q_2, stored in $C_2 = C_2V$
Total quantity, Q, stored $= Q_1 + Q_2$.

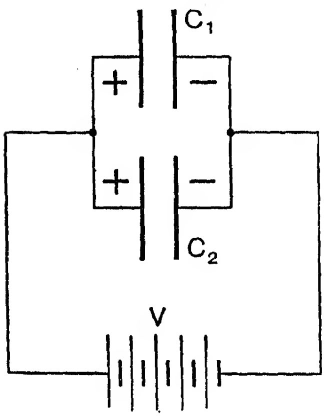

Fig. 50.—Condensers in parallel.

If C_1 and C_2 are replaced by a single condenser of capacitance C so that the same total quantity is stored,

then $Q = CV$
but $Q = Q_1 + Q_2$
hence $CV = C_1V + C_2V$
and $C = C_1 + C_2$.

The joint capacitance of condensers in parallel is therefore given by the sum of their individual capacitances.

Condensers in Series.

In Fig. 51 is shown two condensers, C_1 and C_2, connected in series across a battery of voltage V. This battery voltage will be distributed across C_1 and C_2 such that $V=V_1+V_2$. Since the left-hand plate of C_2 is directly connected to the right-hand plate of C_1, the number of electrons withdrawn from the former must be the same as the number of electrons added to the latter, so that the quantity Q stored in each condenser is the same, irrespective of the relative sizes of the condensers. If the two condensers are imagined to be replaced by a single condenser of capacitance, C, so that the same quantity, Q, is stored, then:

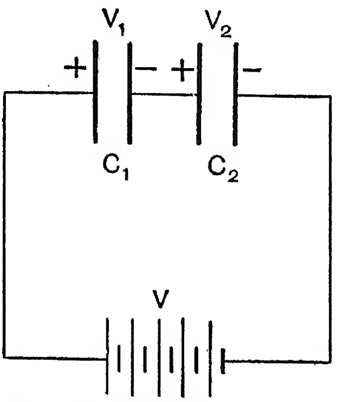

FIG. 51.—Condensers in series.

$$Q = CV \text{ or } V = \frac{Q}{C}$$

$$\text{also} \quad V_1=\frac{Q}{C_1} \text{ and } V_2=\frac{Q}{C_2}$$

$$\text{but} \quad V=V_1+V_2$$

$$\text{hence} \quad \frac{Q}{C}=\frac{Q}{C_1}+\frac{Q}{C_2}$$

$$\text{and} \quad \frac{1}{C}=\frac{1}{C_1}+\frac{1}{C_2}$$

The reciprocal of the joint capacitance of condensers in series is given by the sum of the reciprocals of their individual capacitances.

For two condensers *only*, the joint capacitance is given $\frac{C_1C_2}{C_1+C_2}$.

Example. Two condensers have capacitances of 2 and 4 microfarads respectively. Calculate the joint capacitance (*a*) in parallel, (*b*) in series.

(*a*) $$C=2+4=6 \text{ microfarads.}$$

(*b*) $$\frac{1}{C}=\tfrac{1}{2}+\tfrac{1}{4}=0{\cdot}5+0{\cdot}25=0{\cdot}75$$

$$\text{and } C=\frac{1}{0{\cdot}75}=1{\cdot}33 \text{ microfarads.}$$

$$\text{Alternatively, } C=\frac{2\times 4}{2+4}=\frac{8}{6}=1{\cdot}33 \text{ microfarads.}$$

Voltage Distribution.

When condensers are connected in series it is of importance to know how the battery voltage, V (Fig. 51), is distributed across the two (or more) condensers.

The joint capacitance, C, is calculated from the expression obtained above and the quantity Q stored in each condenser calculated from $Q=CV$,

$$\text{then } V_1=\frac{Q}{C_1}$$

$$\text{and } V_2=\frac{Q}{C_2}$$

As a check, V_1+V_2 must equal V. It will be noted that the smaller condenser receives the greater share of the voltage and it is of importance to ascertain that the insulator between the plates will withstand this voltage before putting the condenser in circuit.

Example. Three condensers having capacitances of 1, 2 and 4 microfarads are joined in series and connected to a supply of 500 volts.

Calculate the p.d. across each condenser.

Let C represent the joint capacitance,

$$\text{then } \frac{1}{C}=\frac{1}{1}+\frac{1}{2}+\frac{1}{4}=1+0{\cdot}5+0{\cdot}25=1{\cdot}75$$

$$\text{and } C=\frac{1}{1{\cdot}75}=0{\cdot}571 \text{ microfarad.}$$

Let Q represent the quantity stored in each condenser,

$$\text{then } Q=\frac{0{\cdot}571}{10^6}\times 500=0{\cdot}000286 \text{ coulomb.}$$

$$\text{Voltage across } 1\ \mu\text{F condenser}=0{\cdot}000286\times\frac{10^6}{1}=286 \text{ volts.}$$

$$\text{Voltage across } 2\ \mu\text{F condenser}=0{\cdot}000286\times\frac{10^6}{2}=143 \text{ volts.}$$

$$\text{Voltage across } 4\ \mu\text{F condenser}=0{\cdot}000286\times\frac{10^6}{4}=\ 71 \text{ volts.}$$

Capacitance and Dimensions of a Condenser.

From the expression obtained for condensers in parallel it is seen that the total capacitance of two similar condensers is double that of one condenser. The effect, however, of connecting two similar condensers in parallel is simply to double the area of the plates. In a similar manner, the effect of connecting, say, four similar condensers in parallel is to increase the area of the plates by four times and to make the total capacitance four times as great. In general, therefore, it may be stated that the capacitance of a condenser is proportional to the area of the plates.

From the expression obtained for condensers in series it is seen that the total capacitance of two similar condensers is one-half that of one condenser. The effect, however, of connecting two similar condensers in series is to double the thickness of the insulation between the plates connected to the supply voltage. If, say, four similar condensers are connected in series the effect is to increase the thickness of insulation by four times and to reduce the total capacitance to a quarter of that of one condenser.

In general, therefore, it may be stated that the capacitance of a condenser is inversely proportional to the thickness of the insulation between the plates.

Let A represent the area of each plate and t the thickness of the insulation (termed the *dielectric*), then:

$$C\propto\frac{A}{t}$$

Capacitance and Dielectric of a Condenser.

If the capacitance of a condenser with air as the dielectric be measured and then the test repeated using, say, paper as the dielectric between the plates, it will be found that the capacitance has increased. The amount of the increase depends on the material of the dielectric used and the ratio of the capacitance with the material as dielectric to the capacitance with air is termed the *permittivity* (or *dielectric constant* or *specific inductive capacity*) of the dielectric.

Let κ (pronounced "kappa") represent the permittivity of the dielectric, then:

$$C \propto \kappa \times \frac{A}{t}$$

A table showing the permittivities of the common dielectrics met with in practice is given below:

Dielectric	Permittivity
Air	1
Paper	2 to 2·5
Paraffin oil . .	2
Mica	5 to 8

Capacitance of a Condenser.

A condenser with two flat plates is only occasionally met with and then when a very small size is required, say of the order of a few micromicrofarads. Air would be used as the dielectric. For larger sizes a multi-plate condenser

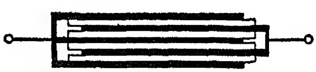

FIG. 52.—Multi-plate condenser.

gives a more convenient shape and this type is illustrated in Fig. 52. A number of metal plates is used, each alternate plate being connected together and brought out to two separate terminals. Adjacent plates are separated by

the insulating medium, usually air or mica. If A is the surface area of one plate, the effective area is A × (number of dielectrics) or, alternatively, A × (N−1), where N is the total number of plates.

$$C=\frac{\kappa A(N-1)}{4\pi t} \text{ electrostatic units.}$$

$$9\times 10^5 \text{ electrostatic units}=1 \text{ microfarad}$$

hence $$C=\frac{\kappa A(N-1)}{113\times 10^5 t} \text{ microfarads.}$$

In the above expression, which is given for reference, A is in sq. cm. and t in cm.

Example. A condenser is made with 21 metal plates. each plate measuring 6 cm. × 5 cm., and separated by sheets of mica having a thickness of 0·2 mm. and a permittivity of 6. Calculate the capacitance in microfarads.

Surface area of one side of each plate $=6\times 5=30$ sq. cm.

$$C=\frac{6\times 30\times 20}{113\times 10^5\times 0{\cdot}02}$$
$$=0{\cdot}016 \text{ microfarad.}$$

Rolled Paper Condenser.

For the larger size condensers paper is invariably employed as the dielectric because of its cheapness and also because it can be obtained in long strips or sheets. It is then common practice to use two thin tin-foil plates of sufficient length to obtain the requisite surface area and to roll the plates and paper sheets to form a convenient cylindrical shape. This is illustrated in Fig. 53 where it will be noted that two paper sheets (shown dotted) are employed, as otherwise the two metal plates would touch after one turn is completed. The effect is to introduce a

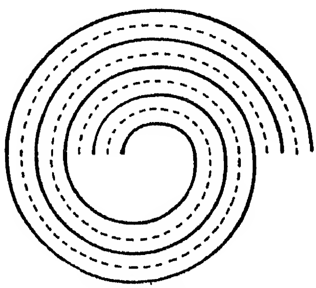

FIG. 53.—Rolle paper condenser

second dielectric and hence the capacitance is double that of a two-plate condenser where the plates are flat.

The thickness of the paper dielectrics will depend on the working voltage for which the condenser is required, but each dielectric will consist of at least two sheets.

Paper condensers are manufactured for working voltages up to several thousands of volts and in sizes up to several microfarads.

Electrolytic Condenser.

This is a type of condenser which has come into use in recent years where the voltage to be applied is unidirectional and not alternating. The great advantage of the electrolytic condenser is the very large capacitance (up to one hundred or more microfarads) which may be obtained in a small space. The two metal plates are of aluminium and are immersed in an electrolyte of ammonium borate. When connected to a d.c. supply a current flows and a film of aluminium oxide is formed on the surface of the plate connected to the positive terminal of the source of supply. Aluminium oxide is a fair insulator and therefore acts as the dielectric, and being extremely thin the capacitance becomes of large value. It will be appreciated that a leakage current (of the order of a milliampere or so) is flowing while the condenser is in circuit and it is essential that the polarity of the supply should not be reversed. In the " wet " type of electrolytic condenser the electrolyte is in liquid form and the condenser should be mounted in a vertical position. In the " dry " type the electrolyte is held in the pores of an absorbent paper and the aluminium sheets are rolled with the absorbent paper, in a similar manner as the paper condenser. Owing to the thin dielectric, electrolytic condensers are only suitable for comparatively low working voltages, up to a few hundreds of volts. The " life " of these condensers is strictly limited due to the evaporation of the electrolyte.

Growth of P.D. across a Condenser.

In Fig. 54 is shown a condenser C in series with a resistance R connected to a battery. An electrostatic voltmeter V is connected across the condenser to indicate the

growth of p.d. At the instant of switching on, there will be no p.d. across the condenser and the whole of the battery voltage will be applied across the resistance, so that the initial value of current will be $\frac{V}{R}$ amperes, where V is the battery voltage. As the condenser charges up, the p.d. across it will rise and will oppose the battery voltage so that if v is the p.d. across the condenser and i the charging current t seconds after switching on, $i=\frac{V-v}{R}$. When the p.d. across the condenser is equal to the applied voltage the current will be zero and the condenser is said to be fully charged. A curve showing the growth of p.d. across the condenser is seen in Fig. 55. OA represents the final voltage to which the condenser is charged and this, of course, is equal to the applied voltage of the battery.

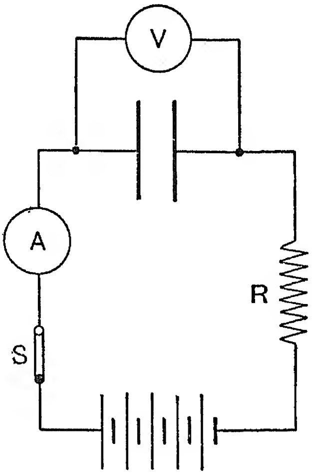

FIG. 54.—Growth of p.d. across a condenser.

If the initial current $\left(\frac{V}{R}\right)$ remained *constant* until the condenser was fully charged to V volts, and if the time taken was T seconds:

$$\text{quantity of electricity} = \frac{V}{R} \times T \text{ coulombs,}$$

$$\text{also, quantity stored} = CV \text{ coulombs,}$$

$$\text{from which, } T = CR \text{ seconds.}$$

This is the time constant of the circuit, and in Fig. 55 AB represents CR seconds, so that the dotted line OB would represent the growth of p.d. if the charging current remained constant. OB is a tangent at O to the actual curve.

Taking any point P on the graph, draw a tangent PD. Then CD would represent the time taken to reach the final value of voltage if charging proceeded from P at a constant rate.

$$PE = v \text{ and } CP = V - v.$$

$$\left.\begin{array}{l}\text{Quantity of electricity in} \\ \quad\text{going from P to D}\end{array}\right\} = i \times CD = \frac{V-v}{R} \times CD \text{ coulombs.}$$

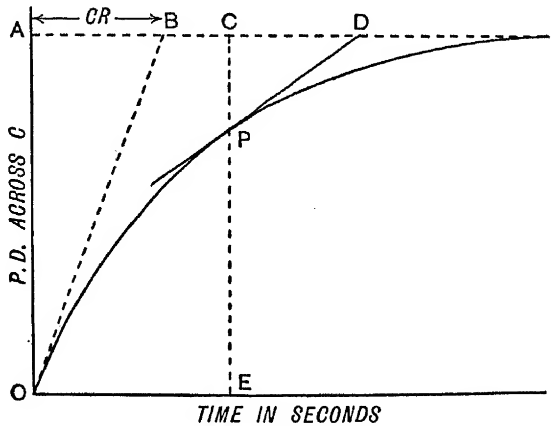

FIG. 55.—Growth of p.d. across a condenser.

Increase in charge on condenser $= C \times (V - v)$ coulombs.

$$\text{hence } \frac{V-v}{R} \times CD = C \times (V-v)$$

$$\text{and } CD = CR \text{ seconds.}$$

This is the time constant, T, of the circuit. Thus, at any point on the curve, as well as at the start, if the growth of p.d. continued at a constant rate it would reach the final value in CR seconds.

Actually, in CR seconds from the instant of switching on, the p.d. across the condenser will have reached 63 per cent. of its final value.

Example. A condenser of 5 microfarads is connected in series with a resistor of 100,000 ohms and a voltage of 500

suddenly applied. What is the time constant of the circuit and the initial value of the charging current ?

$$\text{Time constant} = \frac{5}{10^6} \times 100{,}000$$
$$= 0{\cdot}5 \text{ second.}$$

$$\text{Initial value of current} = \frac{500}{100{,}000}$$
$$= 0{\cdot}005 \text{ ampere}$$
$$\text{or } 5 \text{ milliamperes.}$$

Energy Stored in a Condenser.

When a voltage is applied to a condenser, current flows until the condenser is fully charged. It is clear, therefore, that power and energy has been delivered by the battery and this energy remains stored in the condenser until it is discharged.

Suppose a condenser is charged at a constant rate for t seconds to a final value of voltage V, as illustrated graphically in Fig. 56.

If Q coulombs is the quantity stored, then Q=CV.

$$\text{Charging current} = \frac{Q}{t} = \frac{CV}{t} \text{ amperes.}$$

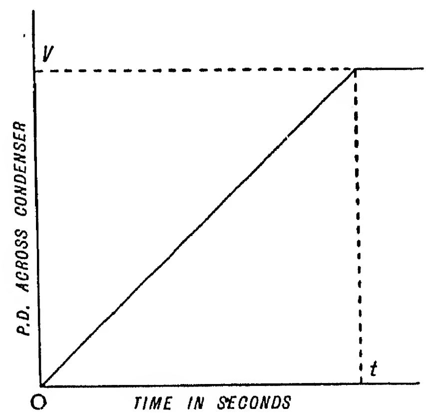

Fig. 56.—Energy stored in a condenser.

Average p.d. across C during charge $=\frac{1}{2}V$ volts.

$$\text{Average power during charge} = \tfrac{1}{2}V \times \frac{CV}{t}$$

$$= \tfrac{1}{2}\frac{CV^2}{t} \text{ watts.}$$

Energy supplied during charge = average power × time

$$= \tfrac{1}{2}\frac{CV^2}{t} \times t$$

$$= \tfrac{1}{2}CV^2 \text{ joules.}$$

Example 1. A condenser of 10 microfarads is charged from a supply of 300 volts. Calculate the energy stored.

$$\text{Energy stored} = \frac{1}{2} \times \frac{10}{10^6} \times (300)^2$$

$$= 0{\cdot}45 \text{ joule.}$$

Example 2. If the condenser in example 1, after disconnection from the supply, is connected in parallel with a condenser of 5 microfarads, what will be the p.d. across the combination and the energy stored?

$$\text{Quantity stored in } 10\ \mu\text{F condenser} = \frac{10}{10^6} \times 300$$

$$= 0{\cdot}003 \text{ coulomb.}$$

This quantity is now shared between the two condensers the total capacitance of the combination being 15 microfarads.

$$\text{P.D. across combination} = \frac{0{\cdot}003}{\frac{15}{10^6}} = 200 \text{ volts.}$$

$$\text{Energy stored in combination} = \frac{1}{2} \times \frac{15}{10^6} \times (200)^2$$

$$= 0{\cdot}3 \text{ joule.}$$

It will be noted that the energy stored is now less. This loss appears as heat in the leads when the circulating current flows on connecting the charged condenser to the uncharged condenser.

CHAPTER VIII

ALTERNATING CURRENTS

Waveform of Generated E.M.F.

When dealing, in Chapter V, with the e.m.f. generated in a coil rotating in a two-pole magnetic field, a graph was given in Fig. 32 illustrating the shape or waveform of the alternating e.m.f. It is seen that the wave is flat-topped and this is largely due to the uniform air-gap producing a uniform flux density under the poles and practically no field between the poles. This is not a convenient shape for alternating voltages and currents as the mathematical law for the graph is complex, if not indeterminate, and the behaviour of inductances and condensers would be difficult to calculate with any degree of accuracy. By lengthening the air-gap gradually, from a minimum at the centre to a maximum at the pole-tips, the density of the magnetic field varies in the air-gap in inverse proportion to the length of gap with the result that the waveform of the generated e.m.f. approximates closely to a *sine* wave. This is a very convenient shape and the graph obeys a simple mathematical law. The alternating current supply of electricity authorities is practically of sine wave shape, or *sinusoidal*, and in most a.c. calculations it is customary to assume that the voltage and current both follow the sine wave law.

Graphical Construction of a Sine Wave.

It is of importance to be familiar with the shape of a sine wave and a simple graphical construction is shown in Fig. 57. A circle is drawn with any convenient radius and radii inserted at intervals of, say, 30°, starting with 0° at the right-hand end of the horizontal diameter. On the right is drawn a horizontal scale marked off in degrees and vertical lines made to intersect the horizontal projections from the ends of the corresponding radii. The curve then drawn through the intersections is a sine wave. For a radius OA drawn at any angle θ (pronounced *thēta*), AB is the value of the alternating quantity corresponding to the

angle θ, since AB is equal to CD. Also, OA is the maximum value, since OA is equal to EF.

For the right-angle triangle OAB,

$$\text{sine } \theta \text{ (written } \sin\theta) = \frac{\text{perpendicular}}{\text{hypotenuse}} = \frac{AB}{OA}$$

$$\text{and} \quad AB = OA \times \sin\theta$$

$$\text{hence} \quad CD = EF \times \sin\theta.$$

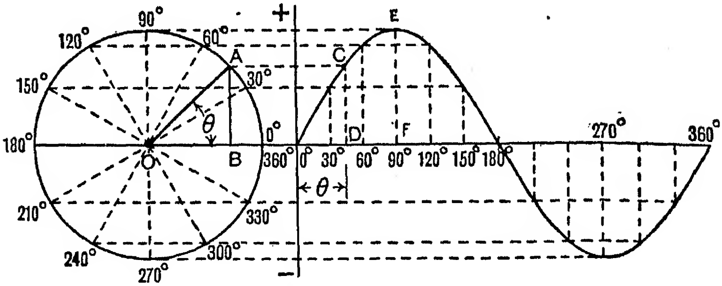

FIG. 57.—Construction of a sine wave.

In words, therefore, the value of an alternating quantity, obeying a sine wave law, corresponding to any angle θ from the start of the wave is given by the maximum value multiplied by $\sin\theta$.

The maximum value is also referred to as the *peak* value and sometimes as the *amplitude* of the alternating quantity.

A sine wave may also be constructed by reference to sine tables and drawing ordinates spaced, say, 30° apart on the horizontal axis.

Thus, $\sin 0° = 0$, $\sin 30° = 0{\cdot}5$, $\sin 60° = 0{\cdot}866$, $\sin 90° = 1{\cdot}0$, etc.

When referring to alternating currents and voltages a complete wave is known as a *cycle* and the number of cycles per second as the *frequency* (symbol f). In the case of a two-pole generator there will be one cycle per revolution so that the frequency is given by the number of revolutions per second. In a four-pole machine there will be two cycles per revolution. In general, let p represent the number of poles and n the speed of the machine in revolutions per second, then:

$$f=\frac{pn}{2} \text{ cycles per second.}$$

If the frequency of an alternating quantity is known, it is clear that a sine wave may also be plotted to a base of time since one cycle represents an interval of time of $\frac{1}{f}$ second. For example, if the frequency of an alternating quantity is 50 cycles per second, 360° corresponds to $\frac{1}{50}$ second, 180° to $\frac{1}{100}$ second, 90° to $\frac{1}{200}$ second, 60° to $\frac{1}{300}$ second, etc.

Equations to a Sine Wave.

In Fig. 58 is shown a sine wave of current plotted to a base marked off both in degrees and in time. At an angle of $\theta°$, corresponding to t second from the start of the cycle,

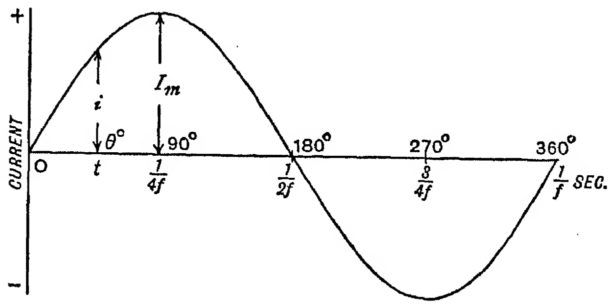

FIG. 58.—Sine wave of current.

the length of the ordinate is shown as i amperes. This is referred to as the instantaneous value of the current. The maximum value of the current is shown as I_m amperes and this occurs at 90° or $\frac{1}{4f}$ second and again at 270° or $\frac{3}{4f}$ second.

Now in t second from the start of a cycle the angle moved through is $(360ft)°$, since in $\frac{1}{f}$ second the corre-

sponding angle is 360°. Hence the angle marked as the equivalent of $(360ft)^\circ$ and the equation $i=I_m$ may also be written in the form $i=I_m \sin (360ft)$.

An angle may be measured in units called radia well as in degrees :

360° is the equivalent of 2π radians.

The equation to a sine wave of current may therefo written :

$$i=I_m \sin 2\pi ft$$

$2\pi f$ is denoted by ω (pronounced *omega*), so that equation above is usually expressed in the form :

$$i=I_m \sin \omega t.$$

Similarly, for a sine wave of e.m.f.,

$$e=E_m \sin \omega t,$$

and for a sine wave of p.d.,

$$v=V_m \sin \omega t.$$

Example. A sine wave of alternating current has a value of 5 amperes and a frequency of 50 cycles per sec What will be the instantaneous value of the current (30°, (*b*) at 240°, and (*c*) at $\frac{1}{300}$ second from the sta a cycle ?

(*a*)
$$\begin{aligned} i&=I_m \sin \theta \\ &=5\times \sin 30 \\ &=5\times 0{\cdot}5=2{\cdot}5 \text{ amperes.} \end{aligned}$$

(*b*)
$$\begin{aligned} i&=5\times \sin 240^\circ \\ &=5\times -\sin 60^\circ \\ &=5\times -0{\cdot}866=-4{\cdot}33 \text{ amperes.} \end{aligned}$$

The minus sign indicates that the current has reverse direction.

(*c*)
$$\begin{aligned} i&=I_m \sin 2\pi ft \\ &=5\times \sin (2\pi\times 50\times \tfrac{1}{300}) \\ &=5\times \sin \frac{\pi}{3} \text{ (radians)} \\ &=5\times \sin 60^\circ \\ &=5\times 0{\cdot}866=4{\cdot}33 \text{ amperes.} \end{aligned}$$

Average or Mean Value of a Sine Wave.

The area under the positive half-cycle of current (see Fig. 58) is a measure of the quantity of electricity flowing in one direction and the area under the negative half-cycle is a measure of the quantity flowing in the opposite direction. These two quantities are equal, and therefore the mean or average value of the current is zero. This explains why a moving-coil instrument, which indicates the average current, will not read on alternating currents. The average value of a half-cycle of current, however, will have a definite value and this may be obtained graphically by drawing a number of ordinates at frequent intervals along the base and by measurement, obtaining their arithmetic mean. It will be found that:

$$\text{Average value} = 0{\cdot}637 \times \text{maximum value.}$$

By a mathematical treatment, it can be shown that the average value is $\frac{2}{\pi}$, or 0·637, times the maximum value. It is only occasionally that it is desired to know the average value; usually when dealing with a rectified alternating current.

Effective or R.M.S. Value of a Sine Wave.

When an alternating voltage or current is read on a voltmeter or ammeter, what value is being measured? The voltage and current are varying from instant to instant and it becomes necessary to arrive at a value which gives the same heating effect in a given resistance as a steady direct current through the same resistance. The heating effect at any instant is given by i^2R, where i is the value of the current in amperes and R is the resistance in ohms. A sine wave of current is drawn as in Fig. 59 (*a*) and below (*b*) is drawn a curve where the ordinates represent the squares of the currents. The dotted line in Fig. 59 (*b*) is the average or mean value of these squared ordinates and this is $\frac{1}{2}I_m{}^2$. If I is the value of the *direct* current to give the same heating effect as the alternating current, then:

$$I^2R = \tfrac{1}{2}I_m{}^2R$$

$$\text{or } I = \frac{1}{\sqrt{2}}I_m$$

$$= 0{\cdot}707I_m.$$

This is the *effective* value of the current and is also ref to as the *root-mean-square* (or r.m.s.) value. It means an effective value of current of, say, 10 amperes rises peak value of $\frac{10}{0\cdot707}$ or 14·14 amperes each half-cyc

Similarly, in the case of an alternating voltage effective or r.m.s. value is given by $0\cdot707V_m$, since

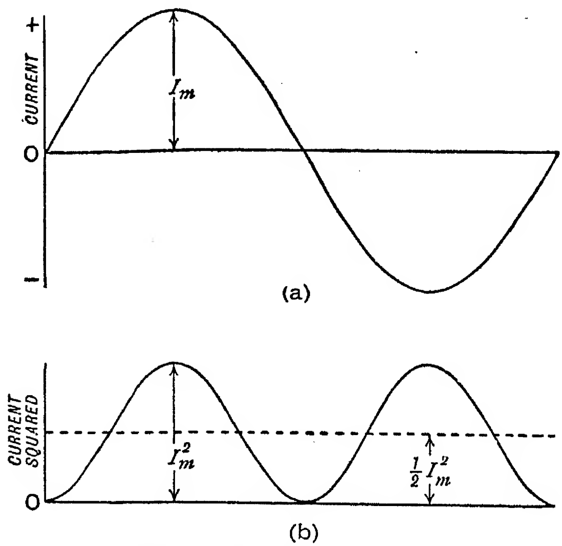

FIG. 59.—Effective value of sine wave of current.

heating effect is proportional to the square of the app voltage and is equal to $\frac{v^2}{R}$ at any instant. Instrument which the deflection varies with the square of the quan to be measured must be used to measure the effective v of an alternating voltage or current and these include hot-wire instrument and the moving-iron instrument.

When the value of an alternating voltage or curren quoted, it is always intended to mean the effective or r. value, unless otherwise stated. The ratio of the effec

value to the average value of an alternating quantity is termed the *form factor* of the wave. For a sine wave:

$$\text{form factor} = \frac{0{\cdot}707 \times \text{maximum value}}{0{\cdot}637 \times \text{maximum value}}$$
$$= 1{\cdot}11.$$

Vectorial Representation of an Alternating Quantity.

In Fig. 60, OA represents to scale the maximum or peak value of an alternating quantity, say, current. Therefore, $OA = I_m$. Suppose OA to rotate about O in a counter-

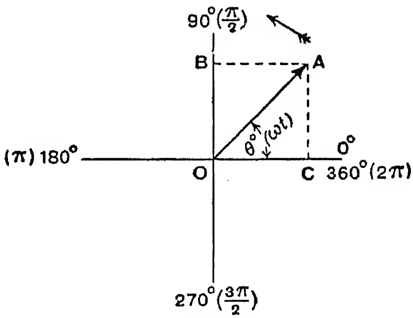

FIG. 60.—Rotating vector.

clockwise direction at a uniform speed of f revolutions per second (f is the frequency of the alternating quantity). The direction of rotation is purely conventional but has been universally accepted. An arrow-head is also placed at the end which rotates; a block arrow-head for a line representing current, such as OA in Fig. 60, and an open arrow-head for a line representing voltage. Such lines are known as *vectors* and this method of representation will be found extremely useful in alternating current problems.

Fig. 60 shows the vector when it has rotated through an angle $\theta°$ (or ωt radians) from the starting-point.

$$OB = AC = OA \sin \theta$$
$$= I_m \sin \theta.$$

But $I_m \sin \theta = i$, namely, the value of the current at that instant. Hence the projection of the vector on the vertical axis gives the instantaneous value of the alternating quantity. Thus, when $\theta=90^\circ$ $\left(\text{or } \frac{\pi}{2} \text{ radians}\right)$, the projection is OA itself, i.e. the current is at its maximum value in a positive direction; when $\theta=180^\circ$ (or π radians), the projection is zero; when $\theta=270^\circ$ $\left(\text{or } \frac{3\pi}{2} \text{ radians}\right)$, the projection is OA again but in a negative direction; and when $\theta=360^\circ$ (or 2π radians) the projection is zero and the vector has completed a cycle. In Fig. 61 (*a*) is shown a wave

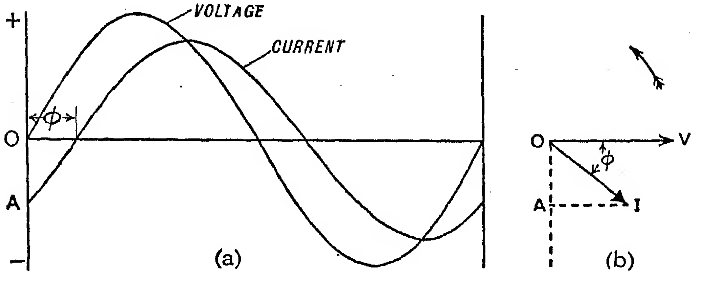

FIG. 61.—Voltage and current differing in phase.

of voltage and a wave of current having the same frequency and drawn to a common base but differing in phase by an angle ϕ. This means that the two waves do not pass through their zero and maximum values at the same instant but are said to have a *phase displacement* of ϕ°. In Fig. 61 (*b*) is shown the corresponding vector diagram, the vectors being stopped at an instant corresponding to zero voltage and an instantaneous value of current given by OA. The current is seen to lag behind the voltage by an angle ϕ. The vector diagram gives all the information required and is a much more convenient illustration than the graphical representation.

Vector Addition.

In a series circuit two or more voltages may have to be added to obtain the overall voltage ; similarly in a parallel circuit two or more currents may have to be added to

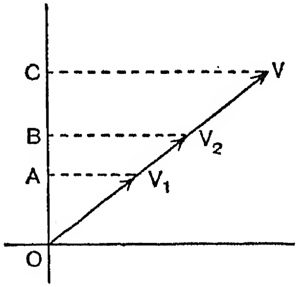

Fig. 62.—Addition of voltages in phase.

obtain the total current. In Fig. 62 is shown two voltages V_1 and V_2 which are in phase, i.e. the voltages pass through the zero and maximum values at the same instants. It is immaterial at which point in a cycle the vectors are

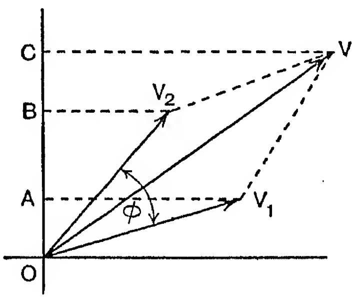

Fig. 63.—Addition of voltages out of phase.

stopped. OA represents the instantaneous value of V_1, OB that of V_2 and OC that of the total voltage V. OC must be equal to OA+OB, from which it follows that $V=V_1+V_2$. This is the same as was found in a d.c.

circuit. In Fig. 63 is shown two voltages V_1 and V_2 with a phase displacement of $\phi°$. Complete the parallelogram OV_1VV_2 and draw in the diagonal OV. It is then clear from the geometry of the figure that $OC = OA + OB$. But OC represents the instantaneous value of OV, and OA and OB the instantaneous values of OV_1 and OV_2 respectively, so that the vector OV represents the maximum value of the resultant voltage to the same scale as OV_1 and OV_2 represent the maximum values of the two separate voltages. OV is termed the *vectorial sum* of the two voltages, and it is evident from the diagram that the resultant or overall voltage is less than the arithmetic sum of OV_1 and OV_2 except when the latter are in phase, as shown in Fig. 62. It is important to remember, therefore, in a.c. work that voltages or currents must not be added arithmetically. Voltmeters and ammeters used in a.c. circuits usually measure the effective (or r.m.s.) value of the voltage and current, so that it is much more convenient when drawing vector diagrams to make the length of the vector represent the effective value rather than the maximum value. This in no way affects the phase relationships between the various quantities, since the effective value of a sine wave is 0·707 times the maximum value and all the vectors are reduced in proportion.

CHAPTER IX

ALTERNATING CURRENT CIRCUITS

Circuit possessing Resistance only.

Consider a circuit having a resistance of R ohms connected across an alternating voltage of sine wave form. At any instant A in Fig. 64 (*a*) the voltage is given by AC, or v volts, and at that instant the current is given by:

$$i = \frac{v}{R} \text{ amperes.}$$

This value of current is AB. When the voltage is zero the current is also zero, and when the voltage is a maximum

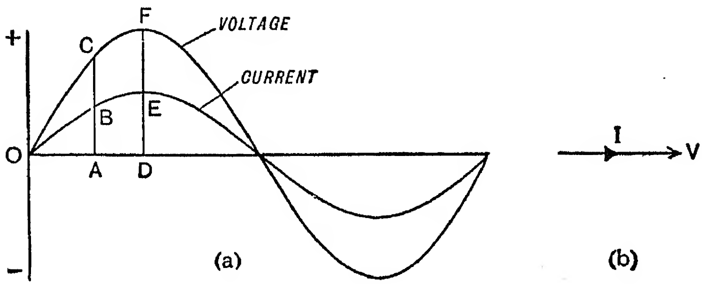

FIG. 64.—Non-inductive circuit.

(DF in the diagram) the current is also a maximum (shown by DE). The two quantities are therefore in phase and the wave of current is also of sine wave shape.

The corresponding vector diagram for a non-inductive circuit is seen in Fig. 64 (*b*).

If V_m and I_m be the maximum values of the voltage and current respectively, then:

$$I_m = \frac{V_m}{R}$$

Let V and I be the effective (or r.m.s.) values, so that:

$$V = 0{\cdot}707\ V_m$$
$$\text{and } I = 0{\cdot}707\ I_m$$

Hence we may write:

$$\frac{I}{0{\cdot}707} = \frac{V}{0{\cdot}707\ R}$$
$$\text{or } I = \frac{V}{R}$$

This is Ohm's law as used in d.c. circuits and can therefore be applied in a.c. circuits containing resistance only.

Circuit possessing Inductance only.

Consider an inductance of L henrys, the resistance of which is so small as to be negligible, connected across an alternating voltage.

The sine wave of current which flows is shown in Fig. 65 (*a*). At instant O the current is growing in a

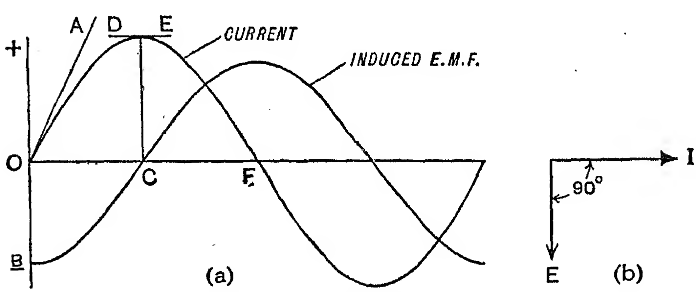

FIG. 65.—Circuit containing inductance only.

positive direction at a rate given by the slope of the tangent OA. This is the maximum slope for the wave and therefore the current is changing at its greatest rate (in amperes per second) when passing through the zero value. Hence the induced e.m.f., given by L × rate of change of current, is at its maximum value in a negative direction, represented by OB in the figure. As the current increases from zero the slope of the curve gets less

and less so that the induced e.m.f. decreases until at instant C, 90° or one quarter-cycle from the start, the slope of the current curve, as represented by tangent DE, is zero and the induced e.m.f. is also zero. As the current decreases between C and F the e.m.f. increases in a positive direction tending, by Lenz's law, to maintain the current flow. By similar reasoning, the complete cycle of induced e.m.f. may be drawn, as shown in Fig. 65 (*a*).

It will be noted that there is a phase displacement of 90°, or one quarter-cycle, between the current and the induced e.m.f. This is readily seen in the vector diagram of Fig. 65 (*b*), the current leading on the induced e.m.f. by 90°.

In order to maintain the flow of alternating current it is clear that a voltage must be applied to the inductance equal and opposite in direction at any instant to the induced e.m.f. This is shown graphically in Fig. 66 (*a*) and vectorially in Fig. 66 (*b*), and from the latter it will be seen that the current lags behind the applied voltage, V, by 90°.

Actually, the applied voltage will be a little greater than the induced e.m.f. because of the resistance of the inductance.

Inductive Reactance.

Referring to Fig. 66 (*a*), the current increases from zero to its maximum value I_m in a quarter of a cycle, or $\frac{1}{4f}$ second.

Average rate of change of current during a quarter of a cycle

$$=\frac{I_m}{\frac{1}{4f}}=4fI_m \text{ amperes per second.}$$

$$\text{Average e.m.f. induced}=L\times 4fI_m \text{ volts.}$$

Average value of applied voltage$=4fLI_m$ volts.

But average value of voltage

$$=\frac{2}{\pi}\times\text{maximum value of voltage}$$

$$=\frac{2}{\pi}\times V_m$$

Hence $\frac{2}{\pi}V_m=4fLI_m$

and $\frac{V_m}{I_m}=2\pi fL.$

If V and I denote the effective values of the applied voltage and current respectively :

$$V=0{\cdot}707\ V_m \text{ and } I=0{\cdot}707\ I_m.$$

Therefore, $\frac{V}{I}=2\pi fL.$

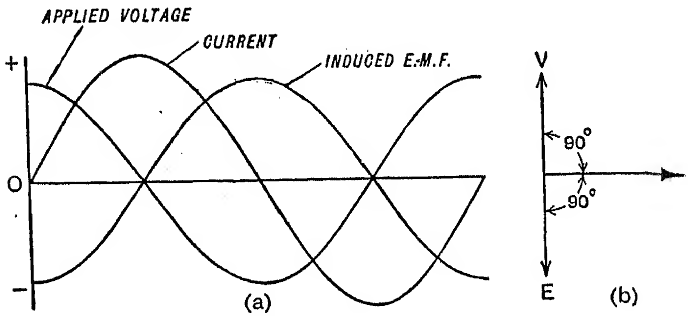

FIG. 66.—Circuit containing inductance only.

$2\pi fL$ (or ωL) is called the *inductive reactance* and is expressed in ohms. Reactance is denoted by the symbol X. The above relationship may be re-written thus :

$$I=\frac{V}{\omega L}=\frac{V}{X}.$$

It is clear that the inductive reactance is directly proportional to the frequency of the applied voltage and this is shown in Fig. 67. The current, for a given voltage, is inversely proportional to the frequency.

Example 1. A coil has an inductance of 200 microhenrys and negligible resistance. What will be its reactance at a frequency of one megacycle per second ?

$$\text{Inductive reactance} = 2\pi f L$$
$$= 2\pi \times 10^6 \times \frac{200}{10^6}$$
$$= 1256 \text{ ohms.}$$

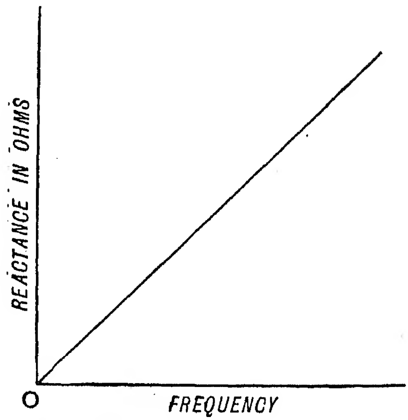

FIG. 67.—Inductive reactance and frequency.

Example 2. An alternating p.d. of 100 volts is applied to a coil of inductance 5 henrys and negligible resistance. What will be the current:

(*a*) at a frequency of 50,
(*b*) at a frequency of 5000?

(*a*) Reactance $= 2\pi \times 50 \times 5$
$= 1570$ ohms

$$\text{Current} = \frac{100}{1570}$$
$= 0{\cdot}063$ ampere
or 63 milliamperes.

(*b*) Reactance $= 157{,}000$ ohms
Current $= 0{\cdot}00063$ ampere
or $0{\cdot}63$ milliampere.

Resistance and Inductance in Series.

Suppose the circuit to consist of a resistance R ohms in series with an inductance L henrys and connected to an alternating p.d. of V volts and frequency f cycles per second. This circuit is shown in Fig. 68 with voltmeters

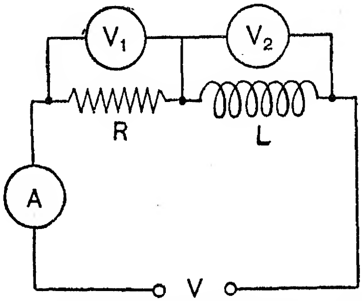

FIG. 68.—Resistance and inductance in series.

connected across R and L, the current being read on the ammeter A.

The vector diagram is seen in Fig. 69. The current I being common to both R and L is taken as the reference vector and is usually drawn horizontally. The voltage, V_1, across R is in phase with I, and the voltage, V_2, across L is drawn leading on I by 90°. The resultant or applied voltage, V, is given by the diagonal of the completed parallelogram. It is seen that the current is lagging behind the applied voltage by an angle ϕ. From the geometry of the diagram:

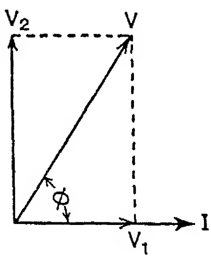

FIG. 69.—Vector diagram for R and L in series.

$$V^2 = V_1^2 + V_2^2$$

but $V_1 = IR$

and $V_2 = IX$, where $X = 2\pi f L$

hence $V^2 = I^2(R^2 + X^2)$

$$V = I \times \sqrt{R^2 + X^2}$$

and $$\frac{V}{I} = \sqrt{R^2 + X^2}$$

$\sqrt{R^2+X^2}$ is termed the *impedance* of the circuit and is expressed in ohms. It is represented by the symbol Z, so that:

$$I=\frac{V}{Z}$$

Impedance Triangle.

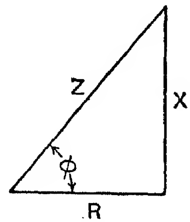

FIG. 70.—Impedance triangle.

Referring to Fig. 70, let the base of the right-angle triangle represent to scale the resistance, R, and the perpendicular the reactance, X, then the hypotenuse, to the same scale, represents the impedance, Z, of the circuit. Further, the angle, ϕ, between the base and hypotenuse gives the phase angle between the applied voltage and current. From the geometry of the triangle:

$$Z=\sqrt{R^2+X^2}$$

$$\text{tangent } \phi \text{ (written } \tan \phi)=\frac{\text{perpendicular}}{\text{base}}=\frac{X}{R}$$

$$\text{cosine } \phi \text{ (written } \cos \phi)=\frac{\text{base}}{\text{hypotenuse}}=\frac{R}{Z}$$

$$\text{sine } \phi \text{ (written } \sin \phi)=\frac{\text{perpendicular}}{\text{hypotenuse}}=\frac{X}{Z}$$

Thus the phase angle may be obtained from any one of the three trigonometrical ratios above.

Example 1. A coil has an inductance of 1 millihenry and a resistance of 5 ohms. What will be its impedance to an alternating current supply having a frequency of 1000 cycles per second and what will be the phase angle of the current with respect to the applied voltage?

$$\text{Reactance } X=2\pi\times 1000\times 0{\cdot}001$$
$$=6{\cdot}28 \text{ ohms.}$$

$$\text{Impedance } Z=\sqrt{R^2+X^2}$$
$$=\sqrt{5^2+6{\cdot}28^2}$$
$$=\sqrt{25+39{\cdot}4}$$
$$=\sqrt{64{\cdot}4}$$
$$=8 \text{ ohms, very nearly.}$$

$$\tan \phi = \frac{X}{R}$$

$$= \frac{6{\cdot}28}{5} = 1{\cdot}26.$$

From the table of trigonometrical ratios the angle wh tangent is 1·26 is 52°, and the current will be lagging this angle behind the applied voltage.

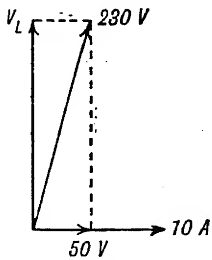

FIG. 71.—Vector diagram for Example 2.

Example 2. Calculate the indı tance of a choking coil to be placed series with a carbon arc taking amperes at 50 volts, in order that t arc may be operated from the 230- 50-cycle supply mains. The resistaı of the coil may be neglected.

The carbon arc may be regard as a non-inductive resistance havi a value of $\frac{50}{10}$ or 5 ohms.

The vector diagram for the circ is given in Fig. 71, where V_L represents the voltage acr the coil.

$$230^2 = 50^2 + V_L{}^2$$
$$V_L{}^2 = 230^2 - 50^2 = 50{,}400$$
$$V_L = 224 \text{ volts}$$

but reactance of coil, $X = \frac{V_L}{I}$

$$= \frac{224}{10}$$

$$= 22{\cdot}4 \text{ ohms.}$$

since $X = 2\pi f L$

$$L = \frac{22{\cdot}4}{2\pi \times 50}$$

$$= 0{\cdot}07 \text{ henry.}$$

Resistance and Inductance in Parallel.

Fig. 72 shows a resistance of R ohms in parallel with an inductance of L henrys connected across an alternating p.d. of V volts. The vector diagram for this circuit is

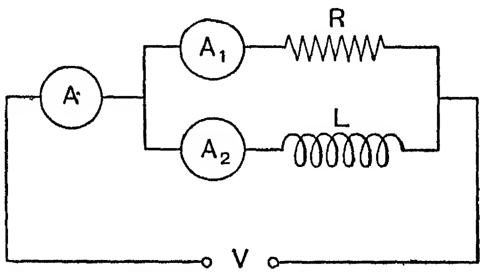

FIG. 72.—Resistance and inductance in parallel.

given in Fig. 73, where it will be seen that the applied voltage V is the reference vector since this quantity is common to both the resistance and inductance. The current, I_1, through the resistance is in phase with the applied

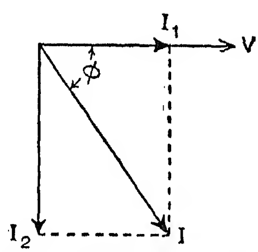

FIG. 73.—Vector diagram for R and L in parallel.

voltage and the current, I_2, through the inductance lags 90° behind V. The resultant current, I, is the vector sum of I_1 and I_2 and is lagging by an angle ϕ behind the applied voltage.

$$I_1=\frac{V}{R}$$

$$I_2=\frac{V}{\omega L}$$

$$I=\sqrt{I_1^2+I_2^2}$$

$$\tan\phi=\frac{I_2}{I_1}$$

$$=\frac{\frac{V}{\omega L}}{\frac{V}{R}}=\frac{R}{\omega L}$$

Example. A resistance of 200 ohms and a coil of inductance 0·1 henry and negligible resistance are connected in parallel across a supply of 10 volts and frequency 500 cycles per second. Calculate: (*a*) the current in each circuit, (*b*) the resultant current, (*c*) the phase angle between the resultant current and the applied voltage; and (*d*) the joint impedance.

(*a*) Current through $R=I_1=\frac{10}{200}=0{\cdot}05$ A.

Current through $L=I_2=\frac{10}{2\pi\times 500\times 0{\cdot}1}=0{\cdot}03$ A.

(*b*) Resultant current $=I=\sqrt{I_1^2+I_2^2}$

from which,

$$I=\sqrt{0{\cdot}05^2+0{\cdot}03^2}$$
$$=\sqrt{0{\cdot}0025+0{\cdot}0009}$$
$$=\sqrt{0{\cdot}0034}$$
$$=0{\cdot}058 \text{ A or } 58 \text{ mA.}$$

(*c*)

$$\tan\phi=\frac{I_2}{I_1}=\frac{0{\cdot}03}{0{\cdot}05}=0{\cdot}6$$
$$\phi=31^\circ \text{ lagging.}$$

(*d*) Joint impedance $=Z=\frac{V}{I}$

from which,

$$Z=\frac{10}{0{\cdot}058}$$
$$=172 \text{ ohms.}$$

CHAPTER X

ALTERNATING CURRENT CIRCUITS
(continued)

Circuit possessing Capacitance only.

The applied voltage to a condenser is shown graphically in Fig. 74 (*a*) and is supposed to obey the law:

$$v = V_m \sin \omega t.$$

If q is the quantity stored at any instant, then $q = Cv$. But the current, i, at the same instant is given by rate of change of q, so that:

$$i = C \times \text{rate of change of voltage.}$$

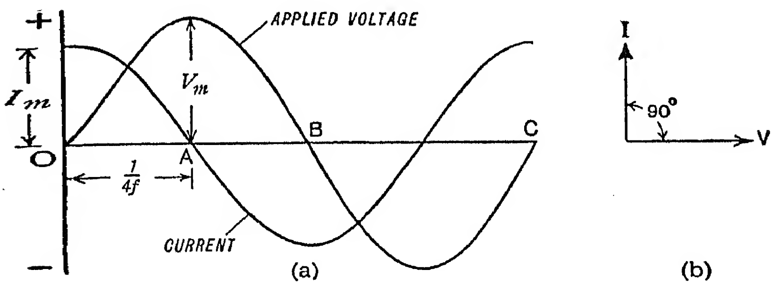

FIG. 74.—Circuit containing capacitance only.

At the instant O, the applied voltage is increasing at the maximum rate, so that the charging current is also at its maximum value I_m. At instant A, a quarter-cycle later, the applied voltage is a maximum, V_m, but its rate of change is zero so that the current is also zero. During the interval of time from A to B the applied voltage is decreasing so that the condenser discharges and the current is in the opposite direction, shown as negative. At instant B the rate of change of voltage is a maximum so that the current is also a maximum. During the interval B to C the variations of applied voltage and current are

the opposite of those for the interval O to B and the cycle is completed at C. Since the condenser charges and discharges there is a to-and-fro movement of electrons to the plates, so that a suitable ammeter connected in the circuit will give a steady reading, viz. the effective value of the current. The vector diagram of Fig. 74 (*b*) shows that the current is leading in front of the applied voltage.

Capacitive Reactance.

At the instant A in Fig. 74 (*a*) the quantity stored in the condenser is CV_m coulombs. This quantity has flowed during the interval O to A, i.e. in $\frac{1}{4f}$ second.

$$\text{Hence average value of current} = \frac{CV_m}{\frac{1}{4f}}$$

$$= 4fCV_m$$

$$\text{but average value of current} = \frac{2}{\pi} \times I_m$$

$$\text{therefore, } \frac{2}{\pi} \times I_m = 4fCV_m$$

$$\text{or } \frac{V_m}{I_m} = \frac{1}{2\pi fC} = \frac{1}{\omega C}.$$

If V and I denote, as usual, the effective values of voltage and current it follows that:

$$\frac{V}{I} = \frac{1}{\omega C}$$

$\frac{1}{\omega C}$ is termed the *capacitive reactance* of the condenser and is expressed in ohms.

It will be seen that the reactance of a condenser is inversely proportional to its capacitance and also to the frequency of the applied voltage. The variation of reactance with frequency is shown graphically in Fig. 75.

Example 1. The current taken by a condenser when connected to 230-V, 50-cycle supply mains is found to

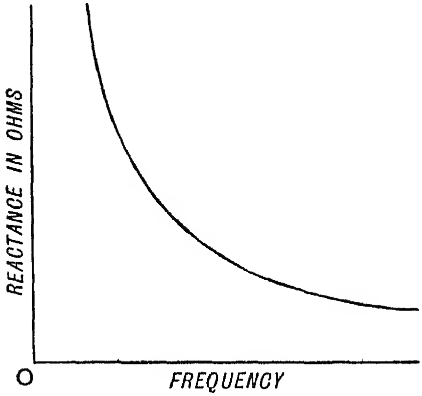

FIG. 75.—Capacitive reactance and frequency.

be 144 milliamperes. Calculate the capacitance of the condenser.

$$\text{Reactance of condenser} = \frac{V}{I}$$

$$= \frac{230}{0 \cdot 144}$$

$$= 1600 \text{ ohms}$$

$$\text{hence} \quad \frac{1}{2\pi fC} = 1600$$

$$\text{and} \quad C = \frac{1}{1600 \times 2\pi f} \text{ farads}$$

$$= \frac{10^6}{1600 \times 2\pi \times 50} \text{ microfarads}$$

$$= 2 \text{ microfarads.}$$

Example 2. Calculate the reactance of a condenser having a capacitance of 0·1 microfarad when connected to a supply frequency of (*a*) 50 cycles per second, (*b*) one megacycle per second.

(a) Reactance$=\dfrac{10^6}{2\pi\times 50\times 0{\cdot}1}$

$=31{,}840$ ohms.

(b) Reactance$=\dfrac{10^6}{2\pi\times 10^6\times 0{\cdot}1}$

$=1{\cdot}59$ ohms.

Resistance and Capacitance in Series.

Fig. 76 shows a resistance of R ohms connected in series with a condenser of C farads to an alternating voltage, V, having a frequency of f cycles per second.

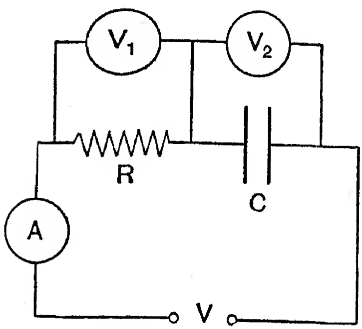

FIG. 76.—Resistance and capacitance in series.

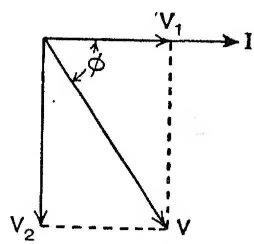

FIG. 77.—Vector diagram for R and C in series.

The p.d.s across the resistance and condenser are V_1 and V_2 volts respectively. In Fig. 77 is given the vector diagram for this circuit. From the geometry of the figure it is seen that :

$$V^2=V_1^2+V_2^2$$

but $V_1=IR$

and $V_2=IX$ where $X=\dfrac{1}{\omega C}$

hence $V^2=I^2(R^2+X^2)$

$$V=I\times\sqrt{R^2+X^2}$$

$\sqrt{R^2+X^2}$ is the impedance, Z, of the circuit and is expressed in ohms, so that :

$$I=\frac{V}{Z}$$

This is exactly the same expression as for resistance and inductive reactance in series. It is important to note, however, that the current is leading on the applied voltage by an angle ϕ and this angle may be obtained from the impedance triangle as before.

Resistance, Inductance and Capacitance in Series.

Fig. 78 shows the circuit and Fig. 79 the corresponding vector diagram for resistance, inductance and capacitance in series.

The voltage, V_3, across the condenser is lagging behind the current by 90° and the voltage, V_2, across the induc-

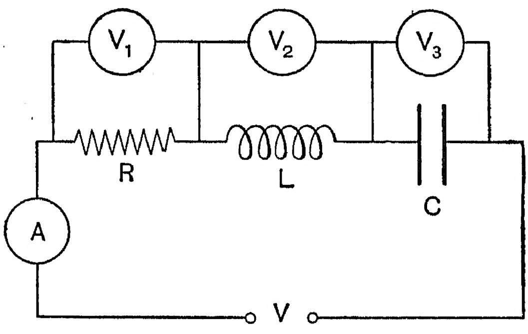

FIG. 78.—R, L and C in series.

tance is leading by 90°. The voltage across the inductance and condenser together is given by V_2-V_3 which is shown as a separate vector in Fig. 79. This voltage when added vectorially to V_1 gives the applied voltage V.

$$V^2 = V_1^2 + (V_2 - V_3)^2$$

$$\text{but } V_1 = IR$$

$$V_2 = I \times \omega L$$

$$\text{and } V_3 = I \times \frac{1}{\omega C}$$

$$\text{hence } V_2 - V_3 = I \times \left(\omega L - \frac{1}{\omega C}\right)$$

$$= IX$$

where X is the total reactance of L and C combined.

Substituting for V_1 and $(V_2 - V_3)$, we have:

$$V^2 = I^2 \times (R^2 + X^2)$$

$$\text{or } V = I \times \sqrt{R^2 + X^2}$$

$$\text{and } \frac{V}{I} = \sqrt{R^2 + X^2}$$

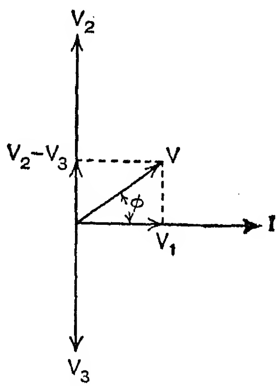

FIG. 79.—Vector diagram for R, L and C in series.

$\sqrt{R^2 + X^2}$ is the impedance, Z, of the circuit and is expressed in ohms. The phase angle, ϕ, may be obtained from the impedance triangle as before. The current may be leading or lagging on the applied voltage according as V_2 or V_3 is the greater, i.e. according as ωL or $\frac{1}{\omega C}$ is the greater. If the inductive reactance is the greater, then

the current will be lagging, as in Fig. 79. If the capacitive reactance is the greater, the current will be leading on the applied voltage.

Example. A coil having an inductance of 0·1 henry and a resistance of 100 ohms is connected in series with a condenser having a capacitance of 0·4 microfarad. If an alternating voltage of 50 and frequency 1000 cycles per second is applied to the circuit, find (*a*) the current (*b*) the p.d. across the coil, (*c*) the p.d. across the condenser, and (*d*) the phase angle of the current.

(*a*) Reactance of coil $=2\pi\times1000\times0{\cdot}1=628$ ohms.

$$\text{Reactance of condenser}=\frac{10^6}{2\pi\times1000\times0{\cdot}4}=398 \text{ ohms.}$$

Total reactance $=628-398=230$ ohms.

Impedance of circuit $=\sqrt{100^2+230^2}$

$=250$ ohms.

$$\text{Current}\quad=\frac{50}{250}=0{\cdot}2 \text{ ampere.}$$

(*b*) The resistance of the circuit is associated with the coil so that it is necessary to obtain the impedance of the coil.

Impedance of coil $=\sqrt{100^2+628^2}$

$=636$ ohms.

p.d. across coil $=0{\cdot}2\times636=127$ volts.

(*c*) p.d. across condenser $=0{\cdot}2\times398=79{\cdot}6$ volts.

(*d*) $\tan\phi=\dfrac{X}{R}$ where X is the total reactance

$$=\frac{230}{100}=2{\cdot}3$$

and $\phi=66°$.

Since the inductive reactance is greater than the capacitive reactance, the current will be lagging behind the applied voltage.

Resonance.

Consider a series circuit consisting of resistance, inductance and capacitance as shown in Fig. 78. The resultant reactance is given by $\left(2\pi f L - \frac{1}{2\pi f C}\right)$, so that if the values of f, L and C are so arranged as to make the inductive reactance equal to the capacitive reactance, the resultant reactance is zero, and the impedance of the circuit is the same as the resistance. In this condition, if the inductive and capacitive reactances are greater than the resistance, the voltages across the coil and condenser will be many times greater than the applied voltage and the circuit is said to be in a state of *resonance*. The vector diagram for a circuit in a resonant condition is given in Fig. 80, where V_2 is the voltage across the inductance, V_3 the voltage across the condenser, and since $V_2=V_3$ the applied voltage V is the same as the voltage, V_1, across the resistance.

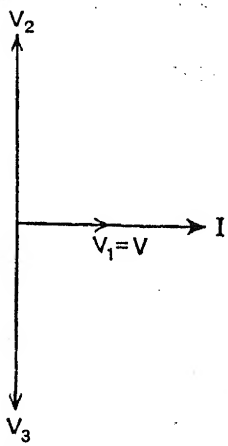

FIG. 80.—Vector diagram for resonant circuit.

$$V_2 = I \times \omega L$$

$$V = V_1 = I \times R$$

$$\text{therefore } \frac{V_2}{V} = \frac{\omega L}{R}$$

This ratio, $\frac{\omega L}{R}$, is known as the *magnification factor* or "Q" factor of the circuit.

Resonant Frequency.

Resonance exists when:

$$\omega L = \frac{1}{\omega C}$$

$$\text{or } \omega^2 LC = 1$$

$$\omega^2 = \frac{1}{LC}$$

$$\text{hence } \omega = 2\pi f = \frac{1}{\sqrt{LC}}$$

$$\text{and} \qquad f = \frac{1}{2\pi\sqrt{LC}}$$

This expression is known as the resonant frequency of the circuit. In Fig. 81 is given a number of instructive curves showing the manner in which the inductive and capacitive

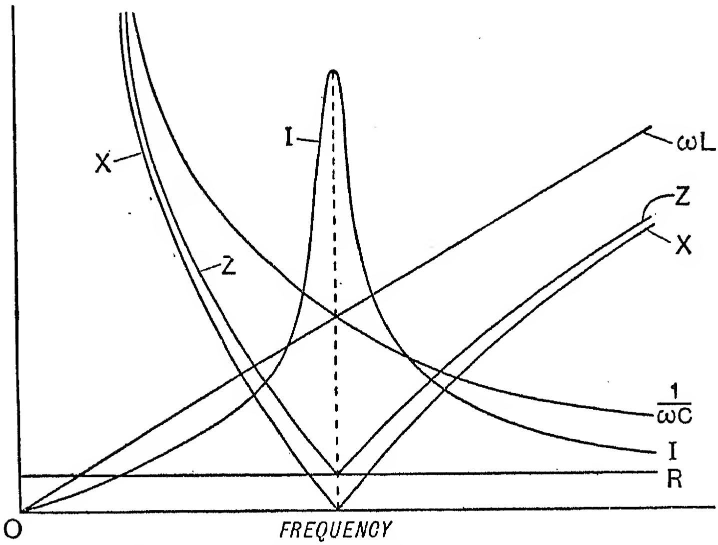

FIG. 81.—Effect of frequency on a series circuit.

reactances and the impedance vary with the frequency of the applied voltage. At the point where the curves of inductive and capacitive reactances cross the total reactance, X, is zero and the impedance, Z, is the same as the resistance.

The graph of current, for a constant applied voltage, shows that this quantity rises steeply to its greatest value at the resonant frequency of the circuit and then falls away rapidly as the frequency is further increased.

Example. A coil of inductance 120 microhenrys and resistance 25 ohms is connected in series with a variable condenser, and a voltage of 0·5 having a frequency of one megacycle per second is applied to the circuit. Find, (*a*) the value of capacitance for resonance, and (*b*) the voltage across the coil at resonance.

(*a*) For resonance,
$$C=\frac{1}{\omega^2 L} \text{ farad}$$
$$=\frac{10^6}{4\pi^2 f^2 L} \text{ microfarad}$$
$$=\frac{10^6\times 10^6}{4\pi^2\times 10^{12}\times 120}$$
$$=0{\cdot}00021 \text{ microfarad}$$
or 210 micromicrofarads.

(*b*) Reactance of coil $=2\pi\times 10^6\times\frac{120}{10^6}=754$ ohms.

Current at resonance $=\frac{0{\cdot}5}{25}=0{\cdot}02$ ampere.

Voltage across coil $=0{\cdot}02\times 754=15$ volts.

Alternatively, using the magnification factor of the coil

Voltage across coil $=0{\cdot}5\times\frac{\omega L}{R}$
$$=0{\cdot}5\times\frac{754}{25}=15 \text{ volts.}$$

Resonance in Parallel Circuits.

Suppose a coil having an inductance of L henrys and a very low resistance R ohms is connected in parallel with a condenser of C farads, as in Fig. 82, and let the frequency of the applied voltage be adjusted until it is the same as the resonant frequency of the closed circuit formed by L, C, and R. The vector diagram for this circuit is shown in Fig. 83 where I_1 is the condenser current leading by 90° on the applied voltage, V, and I_2 is the current through the coil lagging by an angle ϕ on the applied

voltage. As the resistance of the coil is very small compared with the reactance the angle ϕ is nearly 90°.

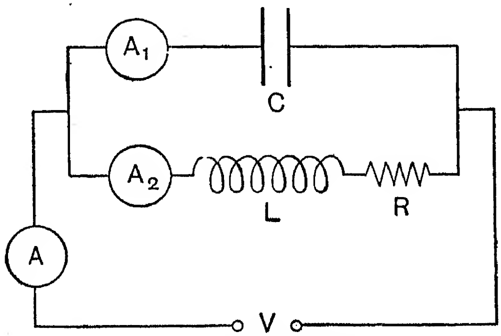

FIG. 82.—Parallel circuits.

In Fig. 83, $OA=I_2\times\sin\phi$

but $$I_2=\frac{V}{\sqrt{R^2+\omega^2L^2}}$$

and $$\sin\phi=\frac{\omega L}{\sqrt{R^2+\omega^2L^2}} \text{ (from Fig. 70)}$$

so that, $$OA=\frac{V\times\omega L}{R^2+\omega^2L^2}$$

$$=\frac{V}{\omega L} \text{ approximately,}$$

since R^2 may be neglected compared with ω^2L^2.

At resonance, $$\omega L=\frac{1}{\omega C}$$

hence $OA=V\times\omega C$.
also, $I_1=V\times\omega C$.

It follows, therefore, that OA and the condenser current I_1 are equal in magnitude, so that the resultant current as read on the ammeter, A, in Fig. 82 is represented by the vector OI in Fig. 83 and is in phase with the applied voltage. The smaller the value of R compared with the

reactance of the coil, the greater will be the angle ϕ and the smaller the resultant current I. It is clear from the vector diagram of Fig. 83 that the currents in the L and C branches may be much greater than the current taken from the supply voltage.

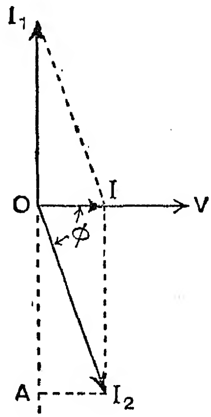

FIG. 83.—Vector diagram for parallel resonant circuit.

$$\frac{OA}{OI}=\tan\phi=\frac{\omega L}{R} \text{ (from Fig. 70).}$$

$$\text{Resultant current}=\frac{OA}{\tan\phi}$$

$$=\frac{\frac{V}{\omega L}}{\frac{\omega L}{R}}$$

$$=\frac{V}{\frac{\omega^2 L^2}{R}}$$

Hence joint impedance at resonance is given by $\frac{\omega^2 L^2}{R}$ ohms.

This is often referred to as the *dynamic resistance* of the

circuit. As the resultant current is in phase with the applied voltage the joint impedance may be considered as the equivalent of a resistance. An alternative expression for the dynamic resistance is obtained as follows:

$$\text{Joint impedance} = \frac{\omega^2 L^2}{R} = \frac{\omega L \times \omega L}{R}$$

$$\text{but } \omega L = \frac{1}{\omega C} \text{ at resonance}$$

$$\text{hence joint impedance} = \frac{1}{\omega C} \times \frac{\omega L}{R}$$

$$= \frac{L}{CR} \text{ ohms.}$$

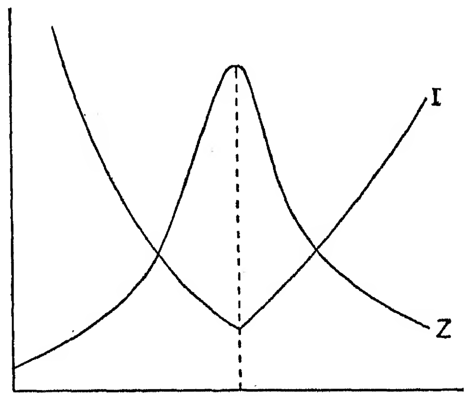

FIG. 84.—Resonance curves for parallel circuit.

In Fig. 84 is shown the manner in which the impedance and current of a parallel circuit vary with the frequency, and it will be seen that the impedance is a maximum at resonance. In the series circuit the impedance is a minimum at resonance.

Example. A coil of inductance 120 microhenrys and resistance 25 ohms is connected in parallel with a variable

condenser, and a voltage of 15 having a frequency of one megacycle per second is applied to the combination. Find (*a*) the value of capacitance for resonance, (*b*) the current in the coil, (*c*) the current in the condenser, (*d*) the resultant current.

(*a*) For resonance,
$$C=\frac{1}{\omega^2 L}\text{ farad}$$
$$=\frac{10^6}{4\pi^2 f^2 L}\text{ microfarad}$$
$$=\frac{10^6\times10^6}{4\pi^2\times10^{12}\times120}$$
$$=0{\cdot}00021\text{ microfarad.}$$

(*b*) Current in coil
$$=\frac{V}{\omega L}\text{ very nearly}$$
$$=\frac{15\times10^6}{2\pi\times10^6\times120}$$
$$=0{\cdot}02\text{ ampere}$$
or 20 milliamperes.

(*c*) Current in condenser
$$=V\times\omega C$$
$$=15\times2\pi\times10^6\times\frac{0{\cdot}00021}{10^6}$$
$$=0{\cdot}02\text{ ampere}$$
or 20 milliamperes.

(*d*) Joint impedance
$$=\frac{L}{CR}$$
$$=\frac{120}{0{\cdot}00021\times25}$$
$$=22{,}850\text{ ohms.}$$

Resultant current
$$=\frac{15}{22{,}850}$$
$$=0{\cdot}00065\text{ ampere}$$
or 0·65 milliampere.

CHAPTER XI

POWER IN THE A.C. CIRCUIT

Power in a Non-inductive Circuit.

It has been shown that when an alternating current with an effective value of I amperes flows through a resistance R ohms, the average power is given by I^2R watts. If V volts be the effective value of the applied voltage to the non-inductive circuit, then $V=IR$.

Average power absorbed $=I^2R=IR\times I$

$$=VI \text{ watts.}$$

Hence the power is given by the product of the voltmeter and ammeter readings, as in the case of a direct current circuit.

Power in an Inductive Circuit.

If a coil be wound with thick wire the resistance may be considered as negligible compared with the reactance and if a voltage, V, is applied to the coil the current is

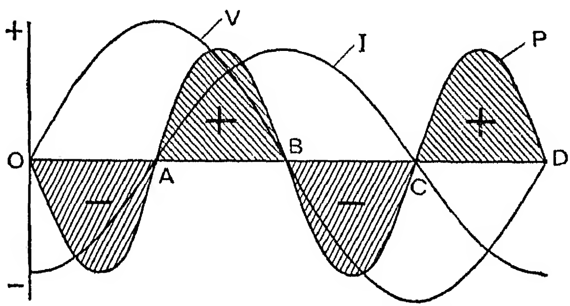

Fig. 85.—Power in inductive circuit.

given by $I=\frac{V}{X}$ amperes and will lag 90° behind the applied voltage. This state of affairs is shown graphically in Fig. 85. The power at any instant is given by the product

of the corresponding instantaneous values of voltage and current. Taking the quarter-cycle from O to A, the current is negative so that the ordinates of the power curve are shown as negative. During the succeeding quarter-cycle from A to B both the voltage and current are in a positive direction, so that the power is shown as positive. From B to C the voltage and power are negative and from C to D both the voltage and current are negative, so that the power is shown as positive over this interval.

It is seen that the shaded areas marked " + " are equal to the areas marked " –," so that the average value of the power over a cycle of voltage and current is zero. We have the somewhat surprising result that no power is absorbed by the coil, although the voltage and current may both be large. The explanation is not difficult. During the quarter-cycles when the current is increasing, energy is being taken from the source of supply (areas marked " + ") and during the quarter-cycles when the current is decreasing, the magnetic field collapses and an equal amount of energy is returned to the supply mains (areas marked " — ").

By exactly similar reasoning it can be shown that the average power absorbed by a condenser is also zero as here, again, the phase displacement between the applied voltage and current is 90° or a quarter-cycle. During the quarter-cycles when the voltage is increasing energy is absorbed from the supply to charge the condenser and during the quarter-cycles when the voltage is decreasing the condenser discharges and an equal amount of energy is returned to the supply.

Power with Resistance and Inductance.

In a circuit containing resistance and inductance the current will lag behind the applied voltage by an angle ϕ and this is shown graphically in Fig. 86. During the interval A to B, the positive area representing the energy absorbed by the circuit is greater than the negative area representing the energy returned to the supply mains during the interval O to A. The area of the power curve is symmetrical about the dotted line EF drawn midway between the positive and negative peaks. The height of EF above the base line OABCD represents the average

power over one cycle of voltage and current. A similar set of curves may be drawn for a circuit containing resistance and capacitance in series. The smaller the phase angle between the voltage and current the greater will be the positive area compared with the negative area and, therefore, the greater the power absorbed by the circuit.

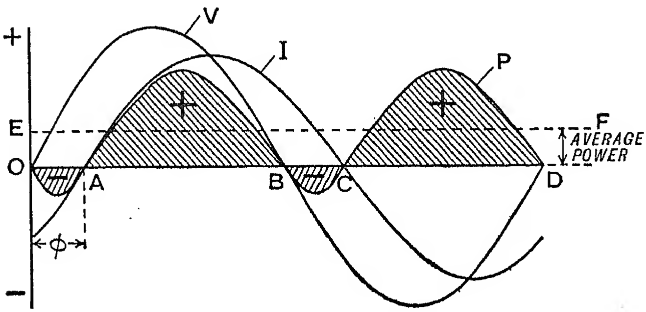

FIG. 86.—Power with R and L in series.

It is now clear that the power absorbed by an a.c. circuit is not given by the product of the voltage and current unless these two quantities are in phase. The product of voltage and current is known as the *voltamperes* of the circuit and this quantity is very often greater than the power, expressed in watts. The number of voltamperes, therefore, has to be multiplied by some factor, equal to or less than unity, to give the power in watts. This factor is termed the *power factor* of the circuit :

Power in watts = voltamperes × power factor.

In any circuit, the power absorbed by that circuit is due to its resistance and is given by I^2R watts. This power gives rise to the heating effect.

$$\text{Power} = I^2R = I \times I \times R$$

but $I = \dfrac{V}{Z}$

and $\text{power} = \dfrac{V}{Z} \times I \times R = VI \times \dfrac{R}{Z}$

VI is the voltamperes

and $\frac{R}{Z}$ is the power factor.

From the impedance triangle, $\frac{R}{Z}=\cos\phi$, where ϕ is the phase angle between the applied voltage and current.

The expression for power in the a.c. circuit may therefore be written:

$$\text{Power}=VI\cos\phi \text{ watts.}$$

For a non-inductive circuit, $\phi=0°$ and $\cos\phi=1{\cdot}0$, so that the power is given by VI watts.

For a purely inductive circuit, or for a condenser only, $\phi=90°$ and $\cos\phi=0$, so that the power is zero.

For a circuit containing R and L, or R and C, where ϕ is, say, 60°, $\cos\phi=0{\cdot}5$ and the power in watts is one-half the voltamperes.

Active and Reactive Components of the Current.

Consider a circuit containing resistance and inductance in series, with the current lagging behind the applied voltage by $\phi°$. The vector diagram for such a circuit is given in Fig. 87.

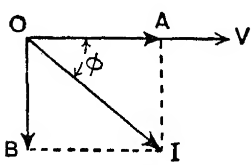

FIG. 87.—Active and reactive components of current.

The current vector OI may be resolved into two components—one, OA, in phase with the voltage, and the other, OB, lagging 90° behind the voltage.

Power in watts $=VI\cos\phi$

but $OA=OI\cos\phi=I\cos\phi$

hence, power $=V\times OA$

OA is termed the *active component* of the current and is equal to $I\cos\phi$. The other component, OB, lags 90° behind the current, so that: power due to a current $OB=V\times OB\times\cos 90°$.

$\cos 90°=0$, and power due to OB is zero.

Hence OB is referred to as the *reactive component* of the current and from Fig. 87 it is seen that:

$$OB=AI=I\sin\phi.$$

Example 1. A coil has an inductance of 0·1 henry and a resistance of 20 ohms. If a voltage of 100 and frequency

50 cycles per second is applied to the coil, show that the power absorbed is given either by I^2R or $VI \cos \phi$.

Also obtain the power factor of the coil.

Reactance of coil $=2\pi fL=2\pi\times 50\times 0{\cdot}1=31{\cdot}4$ ohms.

Impedance of coil, $Z, =\sqrt{20^2+31{\cdot}4^2}=37$ ohms.

Current, I, $=\dfrac{100}{37}=2{\cdot}7$ amperes.

Power $=I^2R=2{\cdot}7^2\times 20=146$ watts.

Power factor $=\cos\phi=\dfrac{R}{Z}=\dfrac{20}{37}=0{\cdot}54$.

Power $=VI\cos\phi=100\times 2{\cdot}7\times 0{\cdot}54$
$=146$ watts.

Example 2. A 1-h.p. motor runs off a 230-V a.c. supply with an efficiency of 75 per cent. and a power factor of 0·7 on full load. Calculate the value of the current.

Power output $=1$ h.p. $=746$ watts

Power input $=746\times\dfrac{100}{75}=995$ watts.

Power $=$ voltamperes $\times$ power factor.

Voltamperes $=\dfrac{995}{0{\cdot}7}=1420$.

Current $=\dfrac{1420}{230}=6{\cdot}2$ amperes.

Measurement of Power and Power Factor.

With many circuits in practice it is not possible to calculate the power factor, and therefore the power cannot be estimated from the voltmeter and ammeter readings. An indicating instrument to measure the average power, called a *wattmeter*, is often employed in a.c. circuits. The wattmeter has two pairs of terminals, one pair being connected as an ammeter and the second pair as a voltmeter. The connections for a wattmeter to measure the power being absorbed by a load is shown in Fig. 88. A voltmeter and ammeter are included so that the product of the readings of these two instruments gives the voltamperes, VI.

$$\text{Power factor} = \frac{W}{VI}$$

Example. The following readings were taken whe testing a coil: voltmeter—100 volts, ammeter—2 ampere wattmeter—150 watts. Find (*a*) the power facto (*b*) phase angle, (*c*) resistance of coil, (*d*) reactance of coi

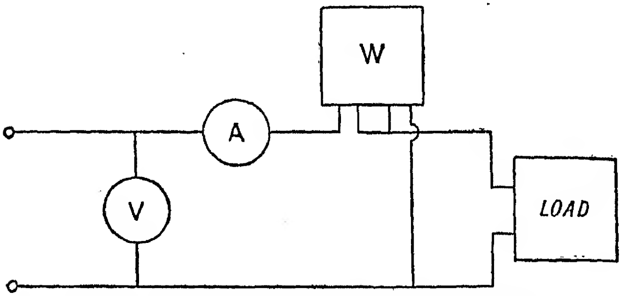

Fig. 88.—Measurement of power and power factor.

(*a*) $\text{Power factor} = \frac{W}{VI} = \frac{150}{200} = 0{\cdot}75$

(*b*) $\text{Power factor} = \cos\phi = 0{\cdot}75$

$\phi = 41°$ lagging.

(*c*) $\text{Power} = I^2R$

and $150 = 2^2 \times R$

hence, $R = \frac{150}{4} = 37{\cdot}5$ ohms.

Alternatively, from the impedance triangle:

$$R = Z \times \cos\phi$$

but $Z = \frac{100}{2} = 50$ ohms

therefore $R = 50 \times 0{\cdot}75 = 37{\cdot}5$ ohms

(*d*) From the impedance triangle:

$$X = \sqrt{Z^2 - R^2}$$
$$= \sqrt{50^2 - 37{\cdot}5^2}$$
$$= 33 \text{ ohms.}$$

CHAPTER XII

THE TRANSFORMER

Principle of the Transformer.

It was seen in Chapter VI that two coils possess mutual inductance when the whole or part of the flux produced by the current in the primary is linked with the turns of the secondary. If the primary carries an alternating current, the flux linking with the secondary will also be alternating and an alternating e.m.f. will be induced in the secondary coil. This is the principle of action of the alternating current transformer and for all frequencies, except the very high frequencies met with in radio practice, the two coils are wound on a common closed iron core so that practically all the flux produced by the primary links with the secondary.

The principle of construction of a transformer is illustrated in Fig. 89. P represents the primary winding, to

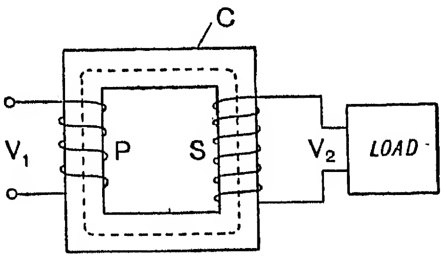

Fig. 89.—The transformer.

which is applied an alternating voltage V_1. S is the secondary winding supplying a voltage, V_2, to the load. C is the closed iron core and the dotted line represents the mean path taken by the flux.

E.M.F. Induced in Secondary.

Let I_1 represent the primary current, and E_2 the secondary e.m.f. :

$$\text{average rate of change of } I_1 = \frac{\text{maximum value of } I_1}{\frac{1}{4f}}$$

$$\text{average value of } E_2 = M \times 4f \times \text{maximum value of } I_1$$

$$\text{but average value of } E_2 = \frac{2}{\pi} \times \text{maximum value of } E_2$$

hence maximum value of E_2

$$= 2\pi f \times M \times \text{maximum value of } I_1$$

and $E_2 = 2\pi f M I_1 = \omega M I_1$

where E_2 = effective value of secondary e.m.f.

and I_1 = effective value of primary current

now $$M = \frac{\Phi_m T_2}{10^8 \times \text{maximum value of } I_1}$$

where Φ_m represents the maximum value of the flux and T_2 the number of secondary turns, so that:

$$\text{maximum value of } E_2 = \frac{2\pi f \Phi_m T_2}{10^8}$$

$$\text{effective value of } E_2 = \frac{0{\cdot}707 \times 2\pi f \Phi_m T_2}{10^8}$$

$$= \frac{4{\cdot}44 f \Phi_m T_2}{10^8}$$

This is the practical form of e.m.f. equation which may also be obtained by the following method:

$$\text{average value of } E_2 = \frac{\Phi_m T_2}{10^8 t}$$

where t is the time interval for one quarter-cycle, viz. $\frac{1}{4f}$ second, so that:

$$\text{average value of } E_2 = \frac{4f \Phi_m T_2}{10^8}$$

$$\text{but effective value of } E_2 = 1{\cdot}11 \times \text{average value of } E_2$$

$$= \frac{4{\cdot}44 f \Phi_m T_2}{10^8}$$

Relationship between Primary and Secondary Voltages and Currents.

As the flux is common to both the primary and secondary the e.m.f. induced *per turn* will be the same for each winding:

hence E_2=e.m.f per turn $\times T_2$

and E_1=e.m.f. per turn $\times T_1$

where E_1 is the primary back e.m.f. and T_1 is the number of primary turns, therefore:

$$\frac{E_2}{E_1}=\frac{T_2}{T_1}$$

In a well-designed transformer the voltage drops in the windings are very small, even on full load, so that the secondary p.d., V_2, is only slightly less than E_2 and the primary applied voltage, V_1, only slightly greater than E_1. For all practical purposes, therefore, we may write:

$$\frac{V_2}{V_1}=\frac{T_2}{T_1}$$

This means that the ratio of the primary and secondary voltages is the same as the ratio of the turns.

$\frac{T_2}{T_1}$ is called the *transformation ratio* and if greater than unity it is a step-up transformer and if smaller than unity a step-down transformer.

The efficiency of a good transformer will vary from 90 per cent. to 99 per cent. at full load, according to the size of the transformer so that we may write with little error:

Output in watts =input in watts

or $V_2I_2\times$power factor$=V_1I_1\times$power factor.

The power factor of the primary will be nearly the same as the power factor of the secondary which is determined by the nature of the load, so that:

$$V_2I_2=V_1I_1$$

and $$\frac{I_2}{I_1}=\frac{V_1}{V_2}=\frac{T_1}{T_2}$$

The ratio of the currents in the two windings is therefore the inverse ratio of the turns.

Expressed in another manner, the ampere-turns on the primary are equal to the ampere-turns on the secondary:

$$I_1T_1 = I_2T_2$$

Example 1. A mains transformer is required to step up the voltage from 230 volts to 500 volts, the frequency being 50 cycles per second. The maximum flux density in the iron core is to be 12,000 lines per sq. cm. and the sectional area of the iron is 10 sq. cms. Calculate the number of turns required on the primary and secondary windings.

The induced e.m.f. in the secondary will be very nearly the same as the required secondary p.d., so that:

$$500 = \frac{4{\cdot}44 f B_m a T_2}{10^8}$$

and $$T_2 = \frac{500 \times 10^8}{4{\cdot}44 \times 50 \times 12{,}000 \times 10}$$

$$= 1880 \text{ turns.}$$

$$\frac{V_1}{V_2} = \frac{T_1}{T_2}, \text{ hence:}$$

Primary turns $$= 1880 \times \frac{230}{500}$$

$$= 865.$$

Example 2. If the transformer in the previous example delivers an output of 1 kW at unity power factor, calculate the currents in the primary and secondary windings.

Secondary current $= \frac{1000}{500} = 2$ amperes

Primary current $= I_2 \times \frac{T_2}{T_1}$

$$= 2 \times \frac{1880}{865} = 4{\cdot}3 \text{ amperes.}$$

Losses in a Transformer.

Copper Loss. Let r_1 represent the resistance of the primary winding and r_2 that of the secondary winding, then the loss is given by:

$$I_1^2 r_1 + I_2^2 r_2 \text{ watts.}$$

This loss is referred to as the copper loss of the transformer and appears as heat in the windings. If the secondary is on open-circuit, i.e. $I_2=0$, then the loss is only $I_1{}^2r_1$ and as I_1 will now be extremely small the loss is negligible for all practical purposes. As the secondary load current approaches the rated output of the transformer the copper loss increases rapidly, being proportional to the square of the current, and at full load represents some 1 per cent. to 5 per cent. of the output.

Iron Loss. The flux in the iron core is alternating and for each cycle of primary current the iron will be taken through a cycle of magnetisation and a loss due to hysteresis will take place. To minimise this loss the area of the

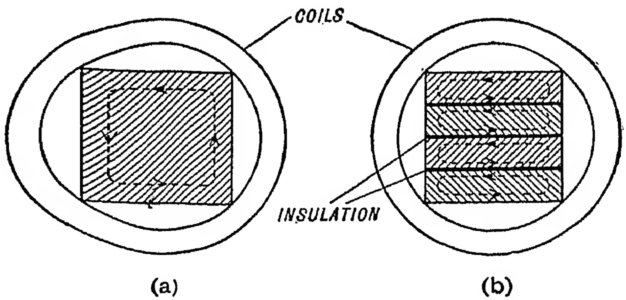

FIG. 90.—Eddy currents.

hysteresis loop is required to be as small as possible and this means the use of a soft iron for the core and also that the maximum flux density shall not greatly exceed the saturation value.

The hysteresis loss is directly proportional to the frequency of the applied voltage to the primary and is independent of the secondary current.

In Fig. 90 (*a*) is shown in section the iron core of a transformer (or choke) and the flux will be passing in a direction at right-angles to the plane of the paper. The core may be looked on as a single turn short-circuited on itself and since the flux links with this turn an e.m.f. will be induced which will cause a current to flow, as shown

of construction and the primary and secondary windings are placed on the limbs A and B, one-half the turns of each winding being placed on each limb. Fig. 92(*b*) shows the *shell type* of construction, the centre limb, D, being

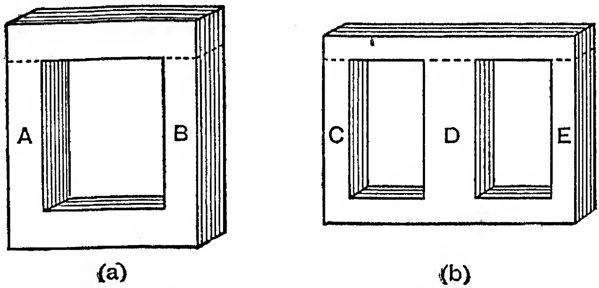

FIG. 92.—Core construction.

twice the sectional area of the outer limbs, C and E. All the turns of both windings are placed on the centre limb which, therefore, carries all the flux, while the outer limbs carry half the flux. The shell type of construction is the more usual arrangement in practice.

Use of Transformer for Impedance Matching.

In Fig. 93 is shown a circuit consisting of a battery of e.m.f. E volts, and internal resistance r ohms, in series

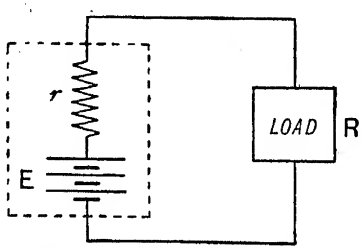

FIG. 93.—Condition for maximum power.

with a load. The internal resistance is shown connected externally for the purpose of illustration. In many cases

it is desired to adjust the resistance of the maximum power is absorbed. If R represents resistance of the load then power absorbed is R.

$$I=\frac{E}{R+r}$$

$$\text{load}=\frac{E^2}{(R+r)^2}\times R$$

shown that this is a maximum when $R=r$, ıe external resistance is equal to the internal the battery. This statement is also applicable of alternating e.m.f. and is true of impedances esistances. It often happens that the internal of the source and the impedance of the load

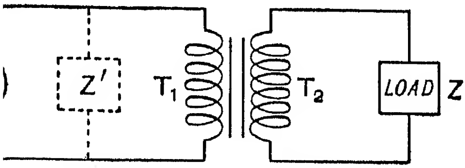

FIG. 94.—Impedance matching.

ed but it is still desired to "match" the two so that maximum power is absorbed by the hing in such cases may be brought about by a transformer and a common example is the e of a radio receiver supplying a loud-speaker Fig. 94 shows an alternating source of e.m.f. ernal impedance of Z' ohms supplying a load e Z ohms through a transformer. The supply onnected to the primary and the load to the To obtain correct matching the load of Z ohms ıt an impedance to the source of supply equal nal impedance, and this is shown dotted as Z'

y current, $I_2,=\frac{V_2}{Z}$

where V_2=secondary voltage.

Primary current, $I_1, = \frac{V_1}{Z'}$

where V_1=voltage of source of supply.

From the above relationships :

$$\frac{I_2}{I_1} = \frac{Z'}{Z} \times \frac{V_2}{V_1}$$

or $$\frac{T_1}{T_2} = \frac{Z'}{Z} \times \frac{T_2}{T_1}$$

from which $$\frac{Z'}{Z} = \frac{T_1^2}{T_2^2}$$

and $$Z' = Z \times \frac{T_1^2}{T_2^2}$$

This means that with, say, a 2 : 1 step-down transform a load of Z ohms connected across the secondary is t equivalent of an impedance of 4Z ohms connected to t voltage supplying the primary.

Example. An output valve with an internal impedan of 4,000 ohms is required to supply a loud-speaker havi an impedance of 10 ohms. What turns ratio must employed for the transformer in order that maximu power may be developed in the speaker ?

$$4{,}000 = 10 \times \frac{T_1^2}{T_2^2}$$

$$\frac{T_1^2}{T_2^2} = \frac{4000}{10} = 400$$

$$\frac{T_1}{T_2} = \sqrt{400} = 20$$

A 20 : 1 step-down transformer must be used.

CHAPTER XIII

INSTRUMENTS AND MEASUREMENTS

Indicating Instruments.

Indicating instruments are employed to measure certain electrical quantities, the reading being given by a pointer moving over a graduated scale. Such instruments include voltmeters, ammeters and wattmeters.

A good voltmeter should have a high resistance so that the current required to produce full-scale deflection is small. A good ammeter should have a low resistance so that the voltage drop in the instrument is small. All the above instruments possess two factors in common:

(*a*) A controlling force, or control, which causes the movement (and pointer) to return to zero when the supply is switched off.

(*b*) A method of damping, so that when the instrument is switched in circuit the tendency of the movement to oscillate is suppressed.

Two forms of control are met with: one a gravity control by means of a weight attached to the spindle, in which type the instrument must be used in a vertical position, and the second a spiral spring control. The latter is similar to the hairspring of a watch and is made of phosphor bronze, the inside end of the spring being attached to the spindle and the outside end to a fixed support. In both forms of control, the controlling force exerted on the spindle of the instrument increases with the deflection.

Damping in an instrument may be effected either by means of a light aluminium piston attached to the spindle and working in a fixed cylinder, known as air damping, or by means of eddy currents set up in an aluminium disc or former attached to the spindle and situated in the magnetic field of a permanent magnet. This latter form is known as magnetic damping.

Classification of Instruments.

There are several types of instruments and the following table summarises these, together with information as to their uses and characteristics. It must be appreciated that the sensitivities given are approximate only.

Type of instrument	Suitable for measuring	Control	Damping	Current for full-scale deflection (voltmeter)	Voltage drop for full-scale deflection (ammeter)
Moving coil	Current and voltage, d.c. only.	Spring	Magnetic	25 μA to 15 mA	6 mV to 75 mV
Moving iron	Current and voltage, d.c. and a.c. (f = 0 to 300).	Gravity or spring	Air	25 mA to 50 mA	0·25 V to 1·0 V
Hot wire	Current and voltage, d.c. and a.c. ($f = 0$ to 30×10^6).	Spring	Magnetic	250 mA	0·5 V
Electrostatic	Voltage only, d.c. and a.c. ($f = 0$ to 2×10^6).	Gravity or spring	Air or magnetic	Zero on d.c.	—
Dynamometer	Current, voltage and power, d.c. and a.c. (f = 0 to 300).	Spring	Air	10 mA to 40 mA	0·5 V to 5·0 V

The Moving-Coil Instrument.

Fig. 95 shows the construction of a moving-coil instrument, the top view representing the elevation and the bottom view a sectional plan through XX. A permanent magnet of "alnico" steel with poles NS is fitted with shaped pole-pieces PP. A soft-iron cylinder I serves to concentrate the flux in the air-gaps and is supported by a plate A of aluminium or brass. The coil, consisting of a number of turns of fine copper wire, silk or enamel insulated, is wound on a thin aluminium former C. SS are two phosphor-bronze hairsprings, the ends of the coil being connected to the inside ends of the springs. The outside

ends of the springs provide the external connections to the coil. When current flows through the turns of the coil, the latter will experience a mechanical force which will produce a deflection and this will be proportional to the value of the current since the magnetic field in the

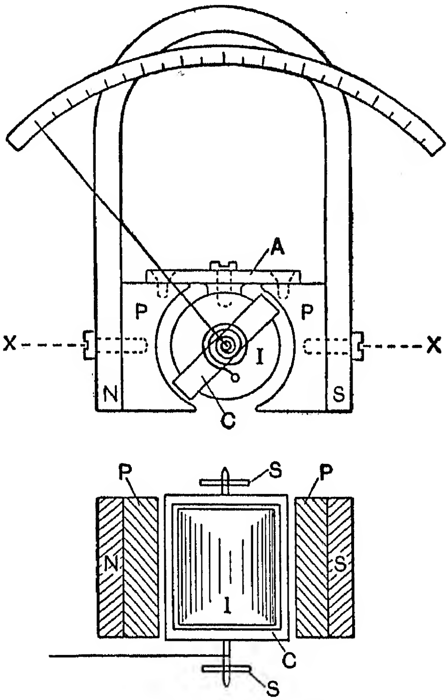

FIG. 95.—Moving-coil instrument.

air-gap is uniform. The divisions on the scale are therefore uniformly spaced. It will be appreciated that the deflection may be either clockwise or anti-clockwise according to the direction of the current through the turns of the coil and the instrument is in consequence said to be polarised. The terminals are therefore marked " + " and

"—" in order to indicate the proper direction of flow to obtain a clockwise deflection. The moving-coil is a most sensitive and accurate instrument and is invariably employed for multi-range test meters, resistors of high value being switched in series to extend the range as a voltmeter and resistors of low value switched in parallel to extend the range as an ammeter. The sensitivity of a moving-coil instrument is often expressed in terms of "ohms per volt": thus a sensitivity of 1000 ohms per volt means that when used as a 0–100 voltmeter the resistance of the instrument is 100,000 ohms, on the 0–200 V range is 200,000 ohms, and so on. A better way to express the sensitivity is the current required to pass through the turns of the coil to produce full-scale deflection and in the above example this would be $\frac{1}{1000}$ ampere or 1 milliampere. A sensitivity of 500 ohms per volt represents a current of $\frac{1}{500}$ ampere or 2 milliamperes for full-scale deflection.

Example. What current will be taken by a 0–200 voltmeter having a sensitivity of 1000 ohms per volt when reading 150 volts?

Resistance of voltmeter $=1000\times200=200{,}000$ ohms.

Current on 150 volts $=\frac{150}{200{,}000}\times1000=0{\cdot}75$ milliampere.

Alternatively:

Current for full-scale deflection $=1$ milliampere.

Current for reading of 150 volts $=\frac{150}{200}\times1=0{\cdot}75$ milliampere.

The Moving-Iron Instrument.

A hollow former F (Fig. 96) is wound with a coil C consisting of a number of turns of insulated copper wire. A strip of soft iron, B, is fixed to the inside of the former and a second strip, A, is attached to an arm fixed to the spindle. When current flows through the turns of the coil a magnetic field is set up inside the former which magnetises

the iron strips A and B with similar poles adjacent so that a force of repulsion takes place and the pointer is deflected. On the current being switched off, the iron strips lose their magnetism and the pointer returns to zero under the influence of the control weight W. Alternatively, a spiral spring may be used as the control, and this is more convenient as the instrument may then be employed in any

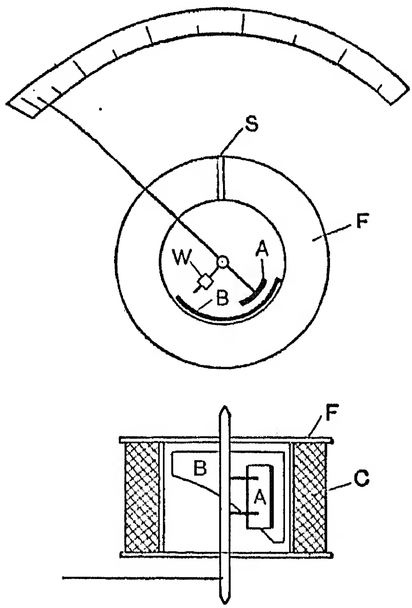

FIG. 96.—Moving-iron instrument.

position. The former is of non-magnetic material but if metallic a saw-cut is made as shown at S in Fig. 96. This prevents the circulation of eddy currents when the instrument is used on a.c.

The scale markings are crowded near the zero end, since the force between A and B is proportional to the current squared, and the instrument is unsuitable for readings below about one-fifth of full-scale reading. When used

as an ammeter the coil is wound with a gauge of wire c sufficient section to carry the current corresponding t full-scale deflection. When used as a voltmeter the co: is wound with a large number of turns of fine gauge coppe wire and a resistor of eureka, or similar wire, having negligible temperature coefficient is placed in series. Th instrument is equally accurate on d.c. and a.c. of lo frequency.

The Hot-Wire Instrument.

Fig. 97 shows the principle of construction of this typ of instrument. W is a wire made of platinum-silver c platinum-iridium alloy stretched between two support

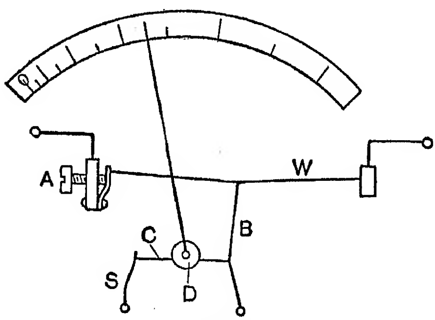

Fig. 97.—Hot-wire instrument.

with the tension adjustable by screw A in order to brin the pointer to zero with no current passing. Attached t W, at or near the centre, is a fine phosphor-bronze wire I fixed at the lower end. Attached to B is a silk thread which passes round a grooved pulley D and is then fixe to a flat leaf-spring S. When current passes through tl wire W, this becomes hot and expands and the elongatic is taken up by B and C through the action of the spring S The pulley D is thereby rotated and the pointer attache to the spindle of D is deflected across the scale. Sin the heating effect is proportional to the square of tl current the scale divisions are crowded near the zero er and open out progressively over the scale. The instrume

is insensitive, with a low accuracy, and is therefore unsuitable for the measurement of very small currents or for use as a voltmeter. It is only found in use for the measurement of high-frequency alternating currents for which the moving-iron instrument is unsuitable. Damping (not shown) is effected by means of an aluminium disc attached to the spindle and situated in the gap between the poles of a permanent magnet.

The Electrostatic Voltmeter.

A force of attraction is exerted between two metallic plates placed close together when one plate is charged positively and the other negatively. This *electrostatic* force of attraction depends on the voltage applied to the plates, the surface area of the plates and the distance apart. If one plate is free to move, a deflection takes place which

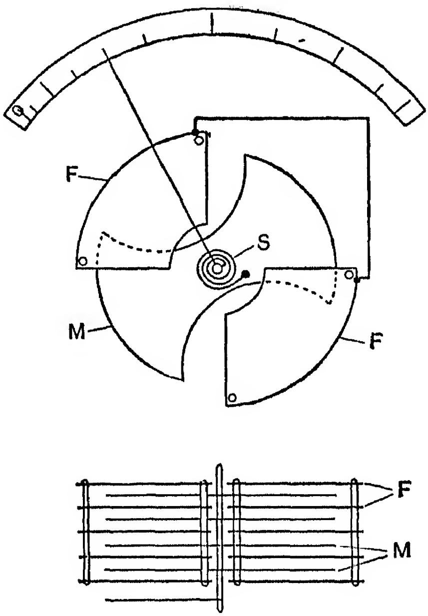

FIG. 98.—Electrostatic voltmeter.

is a measure of the potential difference applied to the plates. The electrostatic voltmeter is the only instrument which measures p.d. directly as other instruments depend for their action on the current produced by the p.d. to be measured.

Except for the initial charging current when switching on, the electrostatic voltmeter takes no current on d.c. and even on a.c. the current is negligible for all but high frequencies. The construction of one form of instrument is shown in Fig. 98, from which it will be seen that a number of fixed (F) and movable plates (M) are employed in order to increase the surface area. The two sets of fixed plates are connected in parallel and the movable plates are attached to a spindle which is controlled by the hair-spring S. The latter provides an electrical connection to the movable plates.

The scale of the electrostatic voltmeter is crowded at the zero end and the first marking is usually one-fifth of full-scale deflection. Industrial type instruments are not normally obtainable for full-scale readings lower than 300 volts. In addition to their use in the laboratory and test-room, electrostatic voltmeters are employed for the measurement of extra high voltages.

The Dynamometer Instrument.

This type of instrument possesses a fixed coil (or coils) and a moving coil, the construction of the latter being identical with that of the coil in the permanent magnet instrument. The moving coil is situated in the magnetic field set up by the fixed coil and the general arrangement of this instrument is shown in Fig. 99 (*a*), the fixed coils, FF, being in section. The moving coil, M, is wound on a light aluminium former and phosphor-bronze springs, S, at top and bottom of the coil, serve both as the control and as connections to the coil. The chief use of the dynamometer instrument is as a wattmeter and the connections are shown in Fig. 99 (*b*). The value of the high resistance, R, in series with the moving coil will depend on the voltage range of the wattmeter, and the gauge and number of turns on the fixed coils will depend on the current range. The load current passes through F and

the current through M is proportional to and in phase with the supply voltage, V. This latter statement is true since the value of R is very high compared with the inductive reactance of M.

Suppose i_F is the instantaneous current through F and i_M the instantaneous current through M, then:

$$\begin{aligned}\text{instantaneous force on } M &\propto i_F \times i_M \\ &\propto i_F \times v\end{aligned}$$

where v=instantaneous p.d. across load. It therefore follows that: instantaneous force on moving coil $\propto$ instantaneous power taken by load, and hence:
average deflecting force on M $\propto$ average value of the power.

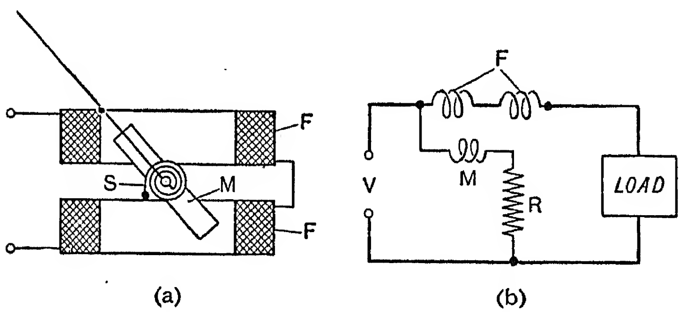

FIG. 99.—Dynamometer instrument and wattmeter connections.

When used as a voltmeter the fixed and moving coils are in series together with a high resistance.

When used as an ammeter the fixed and moving coils are connected in parallel.

The power consumption of the dynamometer instrument is much greater than that of the permanent magnet moving-coil instrument since power is absorbed by the fixed coils providing the magnetic field.

The Ohmmeter.

A scale marked in ohms is often found on the dials of test meters and this apparent direct measurement of

resistance is based on Ohm's law, a dry battery of 3 or 4·5 volts being enclosed in the case of the instrument. The principle of the ohmmeter is seen in Fig. 100, the meter, M, being switched to, say, the 0–1 milliampere range and, simultaneously, a resistance, R, and a battery connected in series. The value of R is so adjusted that with the terminals A and B short-circuited, i.e. when the unknown resistance, X, is zero, full-scale deflection of the meter is obtained and this point is marked " 0 " on the ohms scale. If the value of X, when connected between A and B, is such as to cause half-scale deflection, then X=R. If a quarter-scale deflection is obtained then X=3R, and so

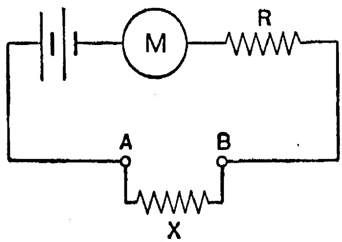

FIG. 100.—The ohmmeter.

on. Thus a scale may be marked off in ohms and this scale is " open " at the zero end and rapidly closes at the other end since it will be appreciated that zero deflection of the meter corresponds to an infinitely high resistance between A and B. The resistance of the meter itself is negligible compared with R and small variations in the p.d. of the battery may be compensated by a variable shunt across the meter. In this manner the pointer is brought to zero on the ohms scale, with A and B short-circuited, before using the instrument as an ohmmeter.

Rectifier Instrument.

The majority of multi-range test instruments include a scale for a.c. volts and amperes although the meter itself is a moving-coil instrument. When switched to the a.c.

range a small copper-oxide rectifier, enclosed in the case of the instrument, is brought in circuit and for each half-cycle of a.c. a unidirectional current passes through the moving-coil meter which reads the mean value of the rectified current.

If the instrument is calibrated with a sine wave of a.c. the scale may be marked in r.m.s. values and is therefore inaccurate if used to measure voltages or currents which are not of sine wave shape.

The rectifier consists of four copper discs, $\frac{1}{2}$ in. to $\frac{3}{4}$ in. diameter, and each disc has a very thin film of oxide on one face. It is found that the disc of copper with a coating

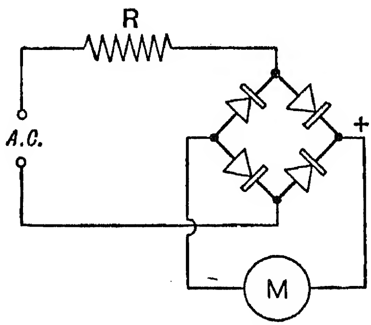

FIG. 101.—Rectifier instrument.

of oxide acts as a rectifier, i.e. it offers a low resistance to the flow of electricity in the direction from oxide to copper and a very high resistance in the opposite direction. The four discs are connected as shown diagrammatically in Fig. 101, each arrow representing a disc and the direction of flow. The rectifier and meter, M, are shown connected for use as a voltmeter, the value of the series resistance, R, being determined by the voltage range.

For use as an a.c. milliammeter or ammeter the rectifier may be shunted, but, preferably, a small transformer, known as a current transformer, should be employed to step down the current by a known ratio, the rectifier and meter being connected to the secondary.

Measurement of Resistance by Voltmeter and Ammeter.

Referring to Fig. 102, X is the unknown resistance of the order of 1 to 50 ohms and B is a battery of suitable p.d. to give as large a value of current as possible without

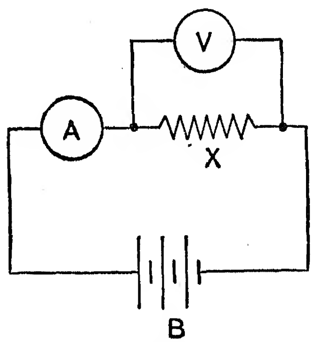

FIG. 102.—Measurement of resistance by voltmeter-ammeter.

producing overheating of the resistance. With the voltmeter connected across X, as shown, the current taken by this instrument also passes through the ammeter, A, but since the current through X is large compared with the voltmeter current the resistance of X for most practical purposes is given by:

$$\frac{\text{reading on V}}{\text{reading on A}}.$$

If it is desired to take into account the voltmeter current, this is calculated by dividing the reading on V by the resistance of V. The actual current through X is then obtained by subtracting the voltmeter current from the reading on A. For the measurement of higher resistances, say from 50 to 1000 ohms, a battery with a somewhat higher voltage is used and the voltmeter is connected as shown in Fig. 103. The voltmeter current is now not read on A and the value of X for practical purposes is again given by:

$$\frac{\text{reading on V}}{\text{reading on A}}.$$

The error now introduced is that the reading on V includes the voltage drop in the ammeter, but as this is small compared with the p.d. across X, the error may usually be neglected. If desired, the voltage drop in A may be

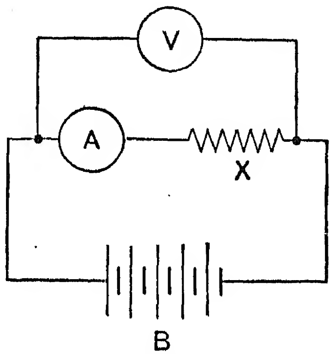

FIG. 103.—Measurement of resistance by voltmeter–ammeter.

calculated by multiplying the reading on A by the resistance of A. This value is then subtracted from the reading on V to give the actual p.d. across X.

Measurement of Resistance by Wheatstone Bridge.

For more accurate measurement of resistances ranging from a fraction of an ohm to several thousands of ohms, the Wheatstone bridge method is invariably employed. The circuit diagram is shown in Fig. 104, where P and Q are two known resistances, R a known resistance variable in steps of 1 ohm from zero to 1110 ohms or 11,110 ohms, G a sensitive centre-zero moving-coil galvanometer and S_1 and S_2 push-button switches. With the unknown resistance, X, connected as shown, R is adjusted until nil deflection is obtained on G on closing the switches S_1 and S_2. There is then no difference of potential between the junctions A and B, and the p.d. across P is the same as the p.d. across R and the p.d. across Q the same as that across X.

Suppose the current through P and Q is I_1 and th current through R and X is I_2:

$$\begin{aligned} &\text{p.d. across } P = I_1P \\ &\text{p.d. across } R = I_2R \\ &\text{hence} \quad I_1P = I_2R \quad \ldots \ldots \quad (1) \\ &\text{p.d. across } Q = I_1Q \\ &\text{p.d. across } X = I_2X \\ &\text{hence} \quad I_1Q = I_2X \quad \ldots \ldots \quad (2) \end{aligned}$$

Dividing (1) by (2) we get:

$$\frac{P}{Q} = \frac{R}{X}$$

and

$$X = R \times \frac{Q}{P}.$$

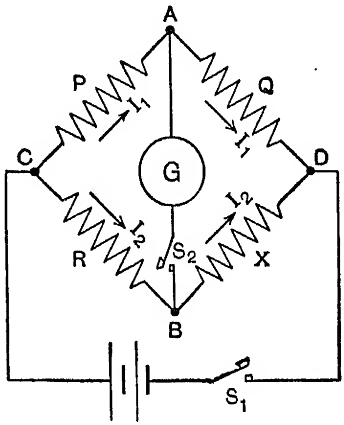

FIG. 104.—Wheatstone bridge.

The resistances P and Q may take the form of a straigl wire of eureka or constantan, in which case R may be fixed value and balance obtained by making the junction (Fig. 104) a sliding contact along the wire. The ratio P to Q will then be the same as the ratio of the lengths the wire. A more convenient and practical method is arrange that P and Q may each be made 1, 10, or 100 ohn and R adjusted for balance. For example, if P=100 an

Q=1 and if R has to be adjusted to 254 ohms for balance to be obtained, then from above relationship :

$$X=254\times\frac{1}{100}=2{\cdot}54 \text{ ohms.}$$

If P and Q had been 1 and 100 ohms respectively and R the same value for balance, then :

$$X=254\times\frac{100}{1}=25{,}400 \text{ ohms.}$$

It will be seen that the Wheatstone bridge makes it possible to measure a very wide range of resistances and to obtain the value of X very readily from the value of R.

In the commercial form of bridge P, Q and R are self-contained in a box and terminals brought out from the junctions A, B, C and D and mounted on a panel together with the switches so that the external connections required are the battery, galvanometer and unknown resistance. The variation of the resistances is brought about by rotary dial switches, with the contact arms and studs mounted on the inside of the panel so as to exclude dust and minimise corrosion.

The Potentiometer.

The principle of this apparatus is shown in the diagram of Fig. 105. AB is a wire of high resistivity and low

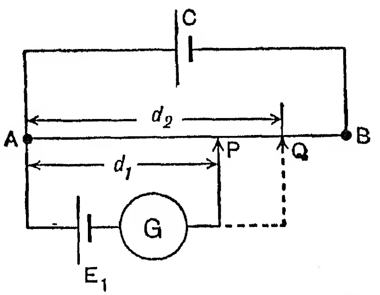

FIG. 105.— Principle of potentiometer.

temperature coefficient (eureka, constantan or manganin) with a uniform diameter and of the order of 1 metre in

length. An accumulator cell, C, of fairly large capacity is connected directly across the wire so that a uniform and steady fall of potential exists from A to B. A standard Weston cadmium cell, E_1, in series with a centre-zero galvanometer, is connected as shown and the sliding contact with the wire adjusted until no deflection takes place on the galvanometer. Under these conditions the difference of potential between A and P on the wire is the same as the e.m.f. of the standard cell (1·0183 volts). The length AP is noted, say d_1. The standard cell is now replaced by a cell or p.d. which is to be measured and a point Q found where no deflection occurs on the galvanometer. The length AQ is noted, say d_2. Then if the unknown e.m.f. or p.d. is represented by E_2, we have the simple relationship :

$$\frac{E_1}{E_2}=\frac{d_1}{d_2}.$$

and

$$E_2=E_1\times\frac{d_2}{d_1}.$$

It will be seen, therefore, that the potentiometer is, in effect, an accurate voltmeter with a very long scale and moreover, when balanced, no current is being taken from the cell or p.d. under test. The maximum voltage which may be measured is limited by the p.d. of the cell, C, and in the simple potentiometer described there is the objection that a calculation is necessary before the unknown voltage is obtained.

Direct-reading Potentiometer.

Referring to the diagram of Fig. 106, the wire, AB, is mounted over a scale divided into, say, 150 parts, and each part divided into tenths. The accumulator cell is connected in series with a variable resistor, R. With the standard cell, E_1, in circuit, the sliding contact is placed at P such that AP represents 101·83 divisions along the scale. The resistor, R, is now adjusted until no deflection is obtained on the galvanometer. The standard cell being replaced by the unknown e.m.f., E_2, the slider is moved until a point Q is found where there is again no deflection. The value of E_2 is then given by AQ divided by 100.

Thus, if balance is obtained at 145·6 divisions, $E_2=1{\cdot}456$ volts. The maximum voltage which may be measured with a wire divided into 150 parts is clearly 1·5 volts,

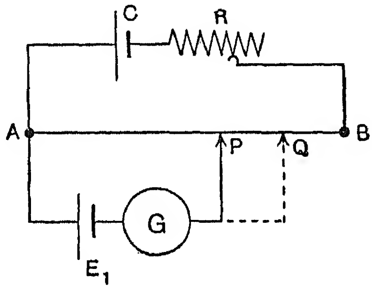

FIG. 106.—Direct-reading potentiometer.

each division representing 0·01 volt and each subdivision 0·001 volt.

The direct-reading potentiometer in its commercial form is met with in test rooms and laboratories for the accurate measurement of d.c. voltage and current, and very low resistances.

Potentiometer Measurement of High Voltages.

To measure voltages greater than the maximum reading on the potentiometer an accurately adjusted potential divider of high resistance, known as a *volt-ratio box*, is

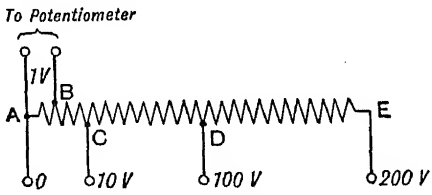

FIG. 107.—Volt-ratio box.

employed, and the circuit for this is shown in Fig. 107. If the voltage to be measured is between 100 and 200 volts this is connected across the terminals marked 0 and 200 V

and the potentiometer is connected to the terminals marked 0 and 1 V. The resistance of the portion AB is exactly $\frac{1}{200}$th of the resistance represented by AE, so that the potentiometer reading multiplied by 200 gives the value of the voltage being measured. The resistance of the portion AD is 100 times the resistance of AB and that of AC 10 times the resistance of AB.

Potentiometer Measurement of Current.

This method is employed for the accurate calibration of ammeters and in Fig. 108, A represents the ammeter to be calibrated and R is a standard resistance of suitable value, the p.d. across which is measured on the potentiometer. The current through the ammeter (and R) may

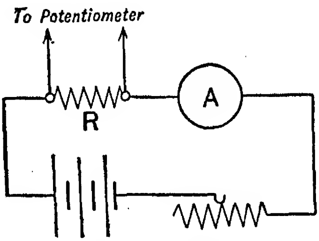

FIG. 108.—Measurement of current by potentiometer.

be adjusted by the variable resistor. If the ammeter to be calibrated has a range of 0–10, then a suitable value for R would be 0·1 ohm and the true value of current is given by the potentiometer reading (in volts) divided by 0·1.

Potentiometer Measurement of Low Resistance.

This method is used where great accuracy is required in the checking of low resistances, such as ammeter shunts, and in Fig. 109 X represents the unknown resistance and R a standard resistance having a convenient value of the same order as X.

A battery of two or three accumulator cells and a variable resistor enable a suitable value of current to be passed through R and X. By means of a two-way switch, S, the potentiometer is connected across R and then across X, and the p.d.s. measured.

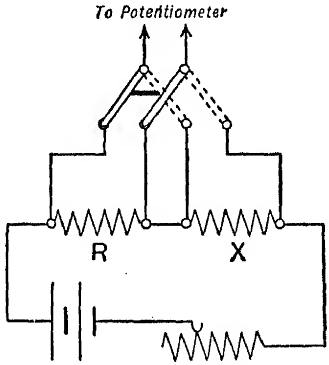

FIG. 109.—Measurement of low resistance by potentiometer.

If V_R represents the p.d. across R and V_X that across X, then :

$$IR = V_R \text{ and } IX = V_X$$

and

$$\frac{X}{R} = \frac{V_X}{V_R}$$

or

$$X = \frac{V_X}{V_R} \times R$$

Example. In the measurement of a resistance by the potentiometer method the p.d. across the standard resistance of 0·1 ohm was 0·562 volt and the p.d. across the unknown resistance 0·7253 volt. Find the value of the unknown resistance.

$$X = \frac{V_X}{V_R} \times R$$

$$= \frac{0{\cdot}7253}{0{\cdot}562} \times 0{\cdot}1$$

$$= 0{\cdot}12905 \text{ ohm.}$$

CHAPTER XIV

ACCUMULATORS

Primary and Secondary Cells.

A cell performs the function of converting chemical energy into electrical energy. In the case of the *primary* cell, when the conversion is completed the cell is of no further use unless and until a fresh supply of chemical ingredients is added. In the case of the *secondary* cell or *accumulator*, when conversion is completed (or partially completed) the chemical ingredients may be restored to their original condition by passing a current through the cell in the opposite direction, a process known as *charging*.

The primary cell in common use is the Leclanché type in which the positive electrode is carbon and the negative electrode is zinc, the electrolyte being ammonium chloride (salammoniac).

When in use, the action of the current through the cell is to produce hydrogen at the positive electrode, the effect of which is to decrease the e.m.f. and to increase the internal resistance of the cell. This effect is known as *polarisation*, and to reduce the accumulation of hydrogen gas a *depolariser* in the form of manganese dioxide (a black powder) surrounds the carbon electrode.

In the "wet" type of cell, the electrolyte is in liquid form, and to prevent mixing, the carbon electrode and manganese dioxide are placed in a porous container. This increases the internal resistance of the wet cell (0·5 to 1·0 ohm), and with depolarisation incomplete the cell is only suitable for the supply of small currents at intermittent periods, such as the working of house-bells and telephones.

In the "dry" type of cell, the electrolyte is in the form of a moist paste, evaporation being retarded by adding zinc chloride to the salammoniac and by sealing the cell (except for a small vent) with bitumen. The porous pot for the carbon electrode and depolariser is now unnecessary,

and with the zinc electrode forming the containing vessel, the internal resistance of the dry cell is less than that of the wet cell, being of the order of 0·1 to 0·3 ohm. Even so, the dry cell is still only suitable for the delivery of small currents (under 100 milliamperes) if required for long periods. The chief use of the small-size dry cell is for the H.T. supply in battery type radio receiving sets.

For the supply of large currents and for continuous working the accumulator must be employed as being a far more economical proposition. The two types of accumulator to be considered are: (*a*) the lead-acid accumulator; (*b*) the nickel-iron or nickel-cadmium alkaline accumulator.

Lead-Acid Accumulator.

The active material on the positive plate of this type of cell is lead peroxide (PbO_2), and on the negative plate is pure lead (Pb). The electrolyte is a dilute solution of sulphuric acid (H_2SO_4) in water. When such a cell is discharged both the lead peroxide and the lead tend to change to lead sulphate ($PbSO_4$), due to the production of hydrogen at the positive plate and oxygen at the negative plate in combination with the acid in the electrolyte. This extraction of acid from the electrolyte causes the latter to become weaker, i.e. the specific gravity is decreased, and this becomes an indication of the state of the cell.

During charge the process is reversed, hydrogen is produced at the negative plate and oxygen at the positive plate, and these gases in combination with the water in the electrolyte convert the lead sulphate back to lead at the negative plate and to lead peroxide at the positive plate. The extraction of water from the electrolyte causes the specific gravity of the latter to rise, and when back to normal is an indication that the cell is fully charged.

It must be understood that the discharge of a cell is discontinued before the whole of the active material has been changed to lead sulphate. An excess of the latter, seen as white flakes in over-discharged and neglected accumulators, is usually a sign that the cell is of no further use.

Construction of Lead-Acid Cell.

The capacity in ampere-hours of a cell will depend the weight of active material which can be supported the plates. This can be increased by using grids of leε antimony alloy cast as shown in section in Fig. 110, a then filling the spaces with a lead compound paste. T paste used for the positive grids is of red-lead (Pb_3O_4) a for the negative grids of litharge (PbO). During 1 initial charge the oxygen produced at the positive conve the red-lead to lead peroxide, which has a dark chocol colour, and the hydrogen produced at the negative redu the litharge to pure lead, which has a slate-grey colo In both cases the active material is now in a porous spongy form, with the result that the electrolyte can mε

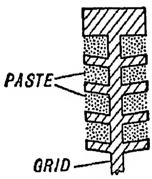

Fig. 110.—Section of pasted plate.

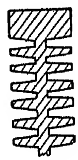

Fig. 111.—Section of formed plate.

contact with the inside as well as the surface of the act material. The disadvantage of the "pasted" pl particularly in the case of the positive, is that for h rates of discharge the expansion of the paste may be s as to cause particles to be thrown out.

This may be overcome by the use of the "forme plate which consists of a moulded plate with deep co gations, as shown in section in Fig. 111. This pl immersed in dilute sulphuric acid, is connected to positive (or negative) of an external source of supply, the production of oxygen (or hydrogen) causes a film lead peroxide to be formed in the case of the positive spongy lead in the case of the negative.

The weight of a formed plate for a given ampere-h capacity is much greater than the pasted plate, so that

use of the formed plate is confined to cells where overall weight is not of great importance, as in stationary batteries.

The capacity of a cell, as already noted, depends on the weight of active material, but the maximum permissible discharge (or charge) current depends also on the construction and arrangement of the plates. For small two-plate cells the capacity is usually based on the 30 or 40-hour rate, which means that the cell will deliver the stated number of ampere-hours in 30 or 40 hours, so that the maximum discharge current is given by the ampere-hours divided by 30 or 40. For most multi-plate cells, the capacity is based on the 10-hour rate, so that the normal charge (and discharge) current is given by: $\dfrac{\text{capacity (Ah)}}{10}$. The arrangement of the plates in a multi-plate cell is shown in plan in Fig. 112, from which it will be seen that an odd

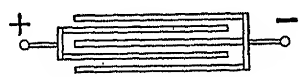

FIG. 112.—Multiplate accumulator.

number of plates is used, there being one more negative plate than the number of positives. The object of this arrangement is to make both sides of each positive plate active and thereby to reduce the tendency of the positives to buckle. Mechanical separators in the form of treated wood strips or perforated sheets are placed between the plates to prevent internal short-circuiting. Ample space is allowed between the bottom of the plates and the base of the containing vessel, the latter being of glass or suitable plastic, or for large stationary cells a lead-lined teak box.

The electrolyte is prepared from pure sulphuric acid and distilled water, and dilution is obtained by adding acid to the water until the required specific gravity is obtained as measured by the reading on a hydrometer. The specific gravity is given by the manufacturer of the cell, and the recommended value for a freshly charged cell usually lies between 1·20 and 1·24. This corresponds approximately to 5 parts of water to 1 part of acid measured by volume.

The strength of the electrolyte affects its resistivity and therefore the internal resistance of the cell, and for specific gravities lying between the values given above the resistivity is a minimum, and this is shown graphically in Fig. 113.

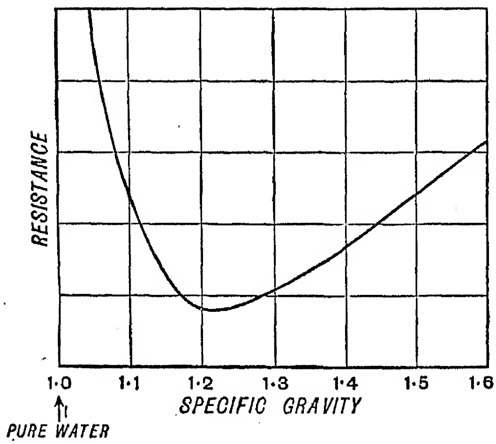

Fig. 113.—Resistance of acid electrolyte.

Charge and Discharge Curves for Lead-Acid Accumulator.

When a cell is discharged, the terminal p.d. falls at a rate which depends upon the discharge current, and discharge should be discontinued when the p.d. falls to about 1·85 volts. This is represented in the discharge curve of Fig. 114, where it is assumed that the discharge current is kept constant and the rated capacity in ampere-hours is delivered in 10 hours. It will be seen that the average p.d. of the lead-acid cell is very nearly 2 volts.

During the charge period, the p.d. applied to the cell has to overcome the e.m.f. of the cell and the voltage drop due to the internal resistance. The charge current is kept constant at the same value as on discharge, and it will be noted that to complete the charge a p.d. of the order of 2·6 to 2·8 volts is required. As the charging period

approaches completion the e.m.f. of the cell is rising, and the hydrogen and oxygen gases with no further chemical work to perform are liberated from the negative and positive plates.

The areas under the curves represent the energies in watt-hours, and the ratio of the energy obtained on dis-

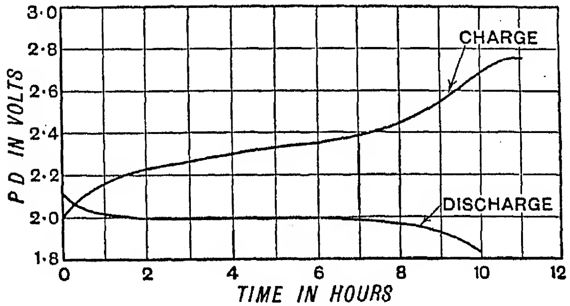

FIG. 114.—Characteristics of ead-acid cell.

charge to that required on charge is the efficiency of the cell. This value for a lead-acid cell is about 75 per cent.

Another efficiency figure, sometimes quoted by manufacturers, is the quantity or ampere-hour efficiency, and this is defined as the ratio of the number of ampere-hours obtained on discharge to that required on charge. This value is about 90 per cent.

Maintenance of Lead-Acid Battery.

To obtain good service and a reasonably long life from a lead-acid battery it is essential that this type of cell be kept under close observation, and the following points, in particular, given attention :

(*a*) Periodical readings should be taken, and recorded, of the specific gravity of the electrolyte.

(*b*) The p.d. of a cell on discharge should not be allowed to fall below 1·8 to 1·85 volts *when delivering normal current.*

(*c*) Charge and discharge rates as specified by the manufacturer should not be exceeded.

(*d*) Cells should not be allowed to stand in an uncharged state; this encourages the growth of the white sulphate.

(*e*) Cells should not be allowed to stand idle, even in a charged state, but should be discharged and charged periodically.

(*f*) The electrolyte should be well above the top of the plates and any evaporation made good by the addition of distilled water.

(*g*) The exterior of the containing vessel should be kept clean and free from electrolyte and any metal part, other than lead, smeared with a thin film of grease.

(*h*) Cells should be given a periodic overcharge at one-half the normal rate until the electrolyte is gassing freely and until there is no further increase in p.d. and specific gravity. This treatment tends to remove any white sulphate and to restore the active material to its normal state.

Alkaline Accumulators.

This type of accumulator is so called because the electrolyte is a solution of potassium hydroxide (caustic potash) in water, having a specific gravity of about 1·17. The electrolyte undergoes practically no change during charge and discharge. In both the nickel-iron and the nickel-cadmium types, the active material on the positive plates is of nickel hydroxide mixed with particles of pure nickel or graphite, the latter being added to reduce the resistance. This material is packed into finely perforated steel tubes and the tubes are then assembled in steel plates or grids. Both the tubes and grids are nickel-plated to prevent corrosion by the electrolyte. In the nickel-iron cell the active material on the negative plates is of iron oxide, and this is packed in perforated pockets and assembled in a similar manner to the positive plates.

In the nickel-cadmium cell the active material on the negative plates is of particles of metallic cadmium mixed with a small quantity of powdered iron. The plates are assembled in nickel-plated sheet-steel containers and are separated from each other and from the container by ebonite strips.

Characteristics of the Alkaline Accumulator.

The charge and discharge curves for an alkaline accumulator are shown in Fig. 115 from which it will be seen that the average p.d. on discharge is about 1·1 volts, compared with 2 volts for the lead-acid cell.

The capacity of an alkaline accumulater is much less affected by the discharge rate than a lead-acid cell. The cell will deliver practically the same number of ampere-hours in 2 hours as in 10 hours without injury to the plates or active material. This means that the cells, if so desired, may be discharged and charged at high current rates. The energy or watt-hour efficiency of

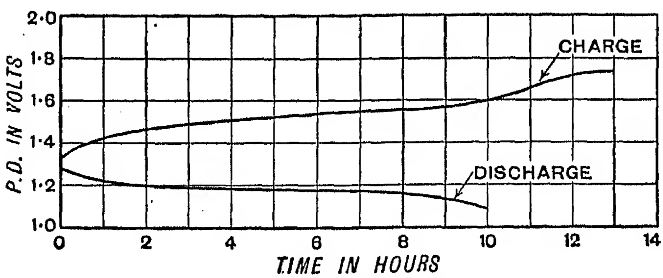

FIG. 115.—Characteristics of alkaline cell.

these cells is about 60 to 65 per cent., and the quantity or ampere-hour efficiency of the order of 75 to 80 per cent.

The advantages of the alkaline accumulator compared with the lead-acid type may be summarised thus:

(*a*) more robust both mechanically and electrically; (*b*) less weight for a given number of watt-hours.

Its disadvantages are: (*a*) a greater number of cells is required for a given voltage owing to the lower average p.d., about 66 per cent. greater than that of lead cells; (*b*) greater cost for a given number of watt-hours; (*c*) greater bulk for a given number of watt-hours.

Example 1. Assuming the terminal p.d. of a lead-acid accumulator varies between 2·1 and 1·85 volts during discharge, calculate the number of cells required to give 100 volts; (*a*) at start of discharge, (*b*) at end of discharge.

$$(a)\ \text{Number of cells} = \frac{100}{2{\cdot}1} = 48$$

$$(b)\ \text{Number of cells} = \frac{100}{1{\cdot}85} = 54.$$

Example 2. A 100-V d.c. supply is available for charging purposes. The following lead-acid cells are required to be charged: (*a*) 10 cells of 60 Ah capacity, (*b*) 30 cells of 30 Ah capacity. If the average p.d. per cell is 2·25 volts, calculate the series resistance required for each group of cells.

(*a*) Charge current (assuming 10-hour rating)

$$= \frac{60}{10} = 6 \text{ amperes.}$$

Charge p.d. $= 10 \times 2{\cdot}25 = 22{\cdot}5$ volts.
Voltage to be "dropped" by series resistance

$$= 100 - 22{\cdot}5 = 77{\cdot}5 \text{ volts.}$$

$$\text{Series resistance} = \frac{77{\cdot}5}{6} = 13 \text{ ohms, very nearly.}$$

$$(b)\ \text{Charge current} = \frac{30}{10} = 3 \text{ amperes.}$$

Charge p.d. $= 30 \times 2{\cdot}25 = 67{\cdot}5$ volts.
Voltage to be "dropped" $= 100 - 67{\cdot}5 = 32{\cdot}5$ volts.

$$\text{Series resistance} = \frac{32{\cdot}5}{3} = 11 \text{ ohms, very nearly.}$$

Example 3. An alkaline cell on discharge delivers 60 ampere-hours, the average terminal voltage being 1·2 volts. On charge, a current of 10 amperes is maintained for 8 hours to bring back the cell to the same state, the average terminal voltage being 1·5 volts.

Calculate: (*a*) the quantity efficiency, (*b*) the energy efficiency.

(*a*) Quantity on charge $= 10 \times 8 = 80$ ampere-hours.

$$\text{Efficiency} = \frac{60}{80} = 0{\cdot}75 \text{ or } 75 \text{ per cent.}$$

(*b*) Energy on discharge $= 60 \times 1{\cdot}2 = 72$ watt-hours.
Energy on charge $= 80 \times 1{\cdot}5 = 120$ watt-hours.

$$\text{Efficiency} = \frac{72}{120} = 0{\cdot}6 \text{ or } 60 \text{ per cent.}$$

UNWORKED EXAMPLES

EXAMPLES ON CHAPTER I

By kind permission of the City and Guilds of London Institute, Department of Technology, a number of questions set in the examinations of that body is reproduced herewith.

1. A current of 2 amperes is allowed to pass through a silver-plating bath for 2 hours. What weight of silver is deposited?

2. It is desired to check the calibration of an ammeter by passing a direct current through the ammeter and a copper sulphate bath. The current is allowed to pass for half an hour and it is found that 3·03 grammes of copper are deposited, the reading on the ammeter being 5 amperes. What is the error of the instrument?

3. Express: (*a*) 0·05 ampere in milliamperes; (*b*) 1500 milliamperes in amperes.

EXAMPLES ON CHAPTER II

1. A battery is formed of 5 cells of the Leclanché type connected in parallel and supplies a current of 1·5 amperes to an external circuit. What is: (*a*) the approximate voltage of the battery, (*b*) the current passing through each cell?

2. Draw a circuit consisting of a plating bath with ammeter and switch, the source of e.m.f. being three cells in series.

3. What do you understand by an *alternating current*?

Giving brief reasons for your answers, state whether such a current can be used—

(i) For electro-plating,
(ii) For heating,
(iii) For charging a secondary battery.

(*C. and G. Tech. Elect., Grade I*, 1940.)

EXAMPLES ON CHAPTER III

1. Two cells, each having an e.m.f. of 2 volts, but having internal resistances of 0·3 ohm and 0·2 ohm respectively, are joined in parallel and are connected across a resistance of 0·68 ohm. What current flows through this resistance, and what current is drawn from each cell? (*C. and G. Tech. Elect., Grade I*, 1942.)

2. A moving-coil instrument has a resistance of 5 ohms and gives a full-scale deflection with a current of 15 milliamperes. Explain how this instrument could be adapted to measure—

(*a*) A current of 5 amperes,
(*b*) A voltage of 50 volts.

3. The resistance of the wire used for a telephone line is 55 ohms per mile when the weight of the wire is 20 lbs. per mile. If the resistivity of the material is 0·77 microhm-inch, what is the cross-sectional area of the wire? What would be the resistance of a loop to a subscriber 5 miles from the exchange if wire of the same material but weighing 70 lbs. per mile were used?

(*C. and G. Tech. Elect., Grade I*, 1941.)

4. The current through an electrical conductor is 1 ampere when the temperature of the conductor is 0° C., and 0·7 ampere when the temperature is 100° C. What would be the current when the temperature of the conductor is 1200° C., and what is the temperature coefficient of resistance of the conductor?

(*C. and G. Tech. Elect., Grade II*, 1942.)

5. A 50,000-ohm and a 100,000-ohm resistance are connected in series across a 200-volt battery. Calculate—

(*a*) The current flowing in the circuit,
(*b*) The watts dissipated by these resistances,
(*c*) The difference in current flow, if in addition a resistance of 25,000 ohms is connected in parallel with the 50,000-ohm resistance.

(*C. and G. Radio Service Work*, 1941.)

EXAMPLES ON CHAPTER IV

1. The following particulars are taken from the magnetic circuit of a relay:

Mean length of iron circuit	20 cm.
Length of air gap	2 mm.
Number of turns on core	8000
Current through coil	50 milliamps.
Ratio B/H for the iron	500

Neglecting leakage, what is the flux density in the air gap?

(*C. and G. Tech. Elect., Grade II*, 1942.)

2. What is the force exerted on a wire 20 cm. long carrying a direct current of 2 amps., when placed in a uniform field of 10,000 lines per sq. cm. whose direction is perpendicular to the wire?

Show by a diagram the direction of this force relative to the directions of current and field.

If the wire were bent back on itself, to form a narrow loop having parallel sides 9 cm. long and 2 cm. apart, what would be the magnitude and direction of the resulting force on the loop? (*C. and G. Tech. Elect., Grade II*, 1941.)

3. What do you understand by the term "magnetic hysteresis"?

Sketch the hysteresis loops of typical samples of soft iron and hard steel and, making reference to your sketches, explain the terms "remanence," "coercive force" and "magnetic saturation." (*C. and G. Tech. Elect., Grade II*, 1938.)

EXAMPLES ON CHAPTER V

1. Explain, as fully as you can, what takes place when a bar magnet is quickly plunged into the centre of a coil of wire and is slowly withdrawn.
(*C. and G. Tech. Elect., Grade I*, 1942.)

2. Explain clearly the function of a commutator in a d.c. generator. The armature of a generator consists of a coil of 20 turns of wire which is wound longitudinally on a drum 10 cm. long and 6 cm. in diameter. If this armature is rotated at a speed of 2000 r.p.m. in a magnetic field of 10,000 lines per square centimetre, what is the average voltage developed across the ends of the coil?
(*C. and G. Tech. Elect., Grade II*, 1939.)

3. A $\frac{1}{2}$-h.p., 230-V., d.c. motor has an efficiency of 75 per cent. Resistance of the armature is 5 ohms. Calculate the current taken by the motor and the back e.m.f. generated.

EXAMPLES ON CHAPTER VI

1. A direct current of 1 ampere is passed through a coil of 5000 turns and produces 10,000 lines of force. Assuming that all lines of force thread all the turns, what is the inductance of the coil? What would be the voltage developed across the coil if the current were interrupted in $\frac{1}{1000}$ second?
(*C. and G. Tech. Elect., Grade II*, 1941.)

2. A battery of 100 volts is suddenly applied to an inductance of 5 henrys and negligible resistance. What would be the value of the current 0·5 second after switching on? What energy is stored in the magnetic field?

3. A rod of ebonite 2·5 cm. square and 30 cm. long is uniformly wound with 200 turns of wire and then bent to form a circle. Calculate the inductance of the coil so formed.

4. A coil carrying 2 amperes and producing 100,000 lines of force is coupled with a second coil of 500 turns so that 75 per cent. of the flux links with it. Calculate the mutual inductance between the coils.

If the current in the first coil is reduced to zero in $\frac{1}{250}$ second, what is the average value of the e.m.f. induced in the second coil?

EXAMPLES ON CHAPTER VII

1. How is an electrical charge "stored" in a condenser, and what happens when the condenser is discharged?

A condenser of 1000 microfarads is charged to a p.d. of 200 volts. If the condenser is discharged in $\frac{1}{1000}$ second, what is the average value of the current during the discharge? (*C. and G. Tech. Elect., Grade I*, 1940.)

2. A parallel plate condenser consists of 11 plates each having an effective area of 100 sq. cm. and spaced 0·02 mm. apart. If the capacitance of the condenser is 0·25 μF, what is the permittivity of the dielectric?
(*C. and G. Tech. Elect., Grade II*, 1941.)

3. Prove that, if two identical condensers are connected in series, the combined capacitance is one-half that of each individual condenser.

Two condensers of 0.02 microfarad and 0·04 microfarad are connected in series across a 100-volt d.c. supply. What would be the voltage developed across each condenser if the insulation resistance of each condenser is (*a*) infinitely great, (*b*) 1 megohm? (*C. and G. Tech. Elect., Grade II*, 1930.)

4. A d.c. supply of 500 volts is applied to a condenser of 30 microfarads in series with a resistance of 1 megohm. What will be the value of the charging current at an instant after switching on equal to the time constant of the circuit?

EXAMPLES ON CHAPTER VIII

1. Compare the heating effect, when flowing in equal resistances, of a direct current of 2 amperes with that of an alternating current having a maximum value of 2 amperes. (*C. and G. Tech. Elect., Grade I*, 1942. *Part question.*)

2. An alternating voltage, defined by the equation $v=100 \sin 100\pi t$, is applied across a resistance of 10 ohms. What is—

(*a*) the frequency of the voltage?
(*b*) the effective (r.m.s.) value of the current?
(*c*) the value of the voltage $\frac{1}{300}$ second from the start of a cycle?

3. In a parallel circuit the two currents of 3 amperes and 5 amperes respectively have a phase displacement of 60°. What is the value of the total current?

EXAMPLES ON CHAPTER IX

1. The operating coil of a relay, having an inductance of 8 millihenrys and a resistance of 30 ohms, is connected across a 5-volt, 800-cycle, a.c. supply. What would be the current through the coil and the phase angle of the current relative to the applied voltage?

(*C. and G. Tech. Elect., Grade II*, 1940. *Part question.*)

2. A moving-iron ammeter, designed to give a full-scale deflection with an alternating current of 5 amperes at a frequency of 50 cycles per second, has an inductance of 0·1 millihenry and negligible resistance. Calculate the value of the non-inductive shunt which is required to permit the ammeter to read up to 15 amps. at the same frequency. (*C. and G. Tech. Elect., Grade II*, 1939.)

3. A circuit consists of a resistance of 100 ohms in series with an inductance. When the frequency of an applied voltage changes from 200 kilocycles to 500 kilocycles per second, the impedance doubles in value. Calculate the value of the inductance. (*C. and G. Radio-Communication, Grade I*, 1943.)

EXAMPLES ON CHAPTER X

1. What values of resistance and capacitance in series would produce the same impedance and phase angle at a frequency of 796 cycles per second ($\omega=5000$) as a resistance of 500 ohms and a condenser of 0·2 μF connected in parallel?

Illustrate your answer by vector diagrams.

(*C. and G. Tech. Elect., Grade II*, 1942.)

2. What do you understand by the term "resonant frequency" of an a.c. circuit?

A coil having an inductance of 1 millihenry and a resistance of 5 ohms is connected in series with a condenser of 10 microfarads. What would be the current flowing through this circuit if an a.c. voltage of 100 volts is applied having a frequency of (i) $\frac{5000}{2\pi}$ cycles per second, (ii) $\frac{10,000}{2\pi}$ cycles per second?

(*C. and G. Tech. Elect., Grade II*, 1939.)

3. A coil of inductance 120 microhenrys and resistance 25 ohms is tuned to resonance at a frequency of 1 megacycle per second by a variable condenser in series. Find the capacitance of the condenser and the "Q" factor of the circuit.

If 1 volt at the resonant frequency is applied to the circuit, what will be the voltage across the coil?

4. An a.c. of 0·5 ampere is passed through a circuit consisting of a coil, having an inductance of 10 millihenrys and a resistance of 10 ohms, which is connected in series with a condenser of 2 microfarads. If the voltage developed across the condenser is 50 volts, what is the voltage developed across the coil?

It is found that, if the capacitance of the condenser is doubled, the current through the coil increases considerably. Why?

Illustrate your answers by vector diagrams drawn approximately to scale.

(*C. and G. Tech. Elect., Grade II*, 1943.)

EXAMPLES ON CHAPTER XI

1. An alternating current of 1 ampere at a frequency of 800 cycles per second flows through a coil the inductance of which is 2·5 millihenrys and the resistance of which is 5 ohms. What is the p.d. across the coil, the power absorbed in the coil, and the power factor? (*C. and G. Tech. Elect., Grade II*, 1941.)

2. A 230-V a.c. motor with an efficiency of 75 per cent. develops 1 h.p. when taking a current of 6·2 amperes. Find the power factor of the motor and the active and reactive components of the current.

EXAMPLES ON CHAPTER XII

1. A small transformer to be used on a 50-cycle supply is required to step up from 230 volts to 500 volts. The iron core available has a sectional area of 6·25 sq. cm. and is to be worked at a flux density of 12,000 lines per square centimetre. Find the number of primary and secondary turns to be wound on the core.

2. An a.c. source with an internal impedance of 10,000 ohms is required to deliver maximum power to a load of 600 ohms. What transformer ratio must be employed for matching ?

3. In the example given on page 127, calculate the maximum efficiency of the transformer and the power output when working at this efficiency.

4. What do you understand by "Hysteresis losses" and "Eddy current losses" in the iron core of a transformer ? State what are the effects of these losses and describe how they may be kept low.

(*C. and G. Tech. Elect., Grade II,* 1939.)

EXAMPLES ON CHAPTER XIII

1. What type of instrument would you employ for the measurement of the following quantities ?

(*a*) An alternating current of 5 amperes with a frequency of 50 cycles per second.
(*b*) An alternating voltage of the order of 20,000 with a frequency of 50 cycles per second.
(*c*) An alternating current of 5 amperes with a frequency of 10^6 cycles per second.
(*d*) A direct current of 10 milliamperes.

2. What current will be taken by a 0–250 moving-coil voltmeter having a sensitivity of 2000 ohms per volt when reading 100 volts ?

3. In a resistance test by the ammeter-voltmeter method the voltmeter, which has a resistance of 600 ohms, is connected across the ends of the resistor. The ammeter reads 1 ampere and the voltmeter 6 volts. What is the percentage error introduced by neglecting the voltmeter current ?

4. Describe the construction and action of an electrostatic voltmeter. What advantages does this type of instrument possess ?

(*C. and G. Tech. Elect., Grade II,* 1941.)

EXAMPLES ON CHAPTER XIV

1. What are the indications that a lead-acid secondary battery has received a full charge ?

A 60-volt, 100 ampere-hour output battery is charged from a 100-volt d.c. supply through a series resistance. What is the approximate cost of one full charge if the battery is in good condition and the cost of the electricity is 1*d.* per kWh ?

(*C. and G. Tech. Elect., Grade II,* 1943.)

2. Why does the specific gravity of a secondary cell vary during the discharge of the cell ?

A telephone repeater station requires a continuous current of 5 amperes at a minimum voltage of 25 volts, which is supplied from a secondary battery. If this battery has to be fully charged in a period of 10 hours every seventh day, what is the required power output and voltage range of the charging generator ?

(*C. and G. Tech. Elect., Grade II,* 1942.)

3. What do you understand by the "ampere-hour efficiency" and the "watt-hour efficiency" of secondary cells ? What are the approximate values obtainable in each case from a large cell in good condition ?

Draw curves, approximately to scale, showing the variation of the voltage of a lead-acid cell during charge and during discharge.

What special arrangements does this variation necessitate when charging the battery ? (*C. and G. Tech. Elect., Grade II,* 1941.)

ANSWERS TO EXAMPLES

CHAPTER I

1. 16·1 grammes. **2.** 0·1 A or 2 per cent. low. **3.** (*a*) 50; (*b*) 1·5.

CHAPTER II

1. (*a*) 1·5 V.; (*b*) 0·3 A.

CHAPTER III

1. Current through 0·68 ohm = 2·5 A. Current through 0·3-ohm cell = 1 A. Current through 0·2-ohm cell = 1·5 A.
2. (*a*) Shunt of 0·015 ohm; (*b*) Series resistor of 3328 ohms.
3. 0·00089 sq. in.; 157 ohms. **4.** 0·16 A; 0·0043.
5. (*a*) 1·33 mA; (*b*) 0·088 W, 0·176 W; (*c*) 1·7 mA.

CHAPTER IV

1. 2100 lines per sq. cm. **2.** 40 grammes; 4 grammes.

CHAPTER V

2. 16 V. **3.** 2·1 A; 219·5 V.

CHAPTER VI

1. 0·5 H; 500 V. **2.** 10 A; 250 joules. **3.** 106 μH. **4.** 0·19 H; 76 V.

CHAPTER VII

1. 200 A. **2.** 5·65. **3.** (*a*) 66·7 V and 33·3 V; (*b*) 50 V across each.
4. 185 μA.

CHAPTER VIII

1. 2 : 1. **2.** (*a*) 50; (*b*) 7·07 A; (*c*) −86·6 V. **3.** 7 A.

CHAPTER IX

1. 0·1 A; 53·3°. **2.** 0·011 ohm. **3.** 92 μH.

CHAPTER X

1. 400 ohms; 1 μF. **2.** (i) 6·3 A; (ii) 20 A. **3.** 0·00021 μF; 30; 30 V.
4. 25 V.

CHAPTER XI

1. 13·5 V; 5 W; 0·37. **2.** 0·7; 4·3 A; 4·4 A.

CHAPTER XII

1. 1380; 3000. **2.** 4 : 1 step-down. **3.** 90·1 per cent.; 457 W.

CHAPTER XIII

2. 0·2 mA. **3.** 1 per cent.

CHAPTER XIV

1. 11 d. **2.** 3kW; 26 V to 30 V.

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